A.D.M. COLLEGE FOR WOMEN (AUTONOMOUS)

(Accredited With 'A' Grade by NAAC 4th Cycle)

(*Affiliated to Bharathidasan University, Tiruchirapalli*)

NAGAPATTINAM – 611 001

PG DEPARTMENT OF PHYSICS

Course Material – Nuclear Physics

(Year – 2023 - 2024)

Prepared by

Dr. N. Lavanya, M.Sc., M.Phil., M.Ed., M.Phil (Edu), PGDEA, Ph.D.,

Assistant Professor of Physics,

A.D.M. College for women,

Nagapattinam.

UNIT – 1 GENERAL PROPERTIES OF NUCLEI AND NUCLEAR MODELS

Constituents of Nuclei:

All atomic nuclei are made up of elementary particles called *protons* and *neutrons.* A proton has a positive charge of the same magnitude as that of an electron. A neutron is electrically neutral. The proton and the neutron are considered to be two different charge states of the same particle which is called a *nucleon.* A species of nucleus, known as a *nuclide*, is represented schematically by ${}_zX^A$ where *Z*, the *atomic number* indicates the number of protons, *A*, the mass number, indicates the total number of protons plus neutrons and *X* is the chemical symbol of the species.

$$N = \text{Number of neutrons} = A - Z.$$

As an example, the chlorine nucleus ${}_{17}Cl^{35}$ has $Z = 17$ protons, $A = 35$ nucleons and $N = 35 - 17 = 18$ neutrons.

Classification of Nuclei

Atoms of different elements are classified as follows :

(i) *Isotopes* are nuclei with the same atomic number *Z* but different mass numbers *A.* The nuclei ${}_{14}Si^{28}$, ${}_{14}Si^{29}$, ${}_{14}Si^{30}$, and ${}_{14}Si^{32}$ are all isotopes of silicon. The isotopes of an element all contain the same number of protons but have different number of neutrons. Since the nuclear charge is what is ultimately responsible for the characteristic properties of an atom, all the isotopes of an element have identical chemical behaviour and differ physically only in mass.

(ii) Those nuclei, with the same mass number *A*, but different atomic number *Z*, are called *isobars.* The nuclei ${}_8O^{16}$ and ${}_7N^{16}$ are examples of isobars. The isobars are atoms of different elements and have different physical and chemical properties.

(iii) Nuclei, with an equal number of neutrons, that is, with the same *N*, are called *isotones.* Some isotones are ${}_6C^{14}$, ${}_7N^{15}$ and ${}_8O^{16}$ ($N = 8$ in each case).

(iv) There are atoms, which have the same *Z* and same *A*, but differ from one another in their nuclear energy states and exhibit differences in their internal structure. These nuclei are distinguished by their different life times. Such nuclei are called *isomeric nuclei* or *isomers.*

(v) Nuclei, having the same mass number *A*, but with the proton and neutron number interchanged (that is, the number of protons in one is equal to the number of neutrons in the other) are called *mirror nuclei.*

Example. ${}_4Be^7$($Z = 4$ and $N = 3$) and ${}_3Li^7$($Z = 3$ and $N = 4$).

Nuclear Mass and Binding Energy

Nuclear mass. We know that the nucleus consists of protons and neutrons. Then the mass of the nucleus should be

$$\text{assumed nuclear mass} = Zmp + Nmn$$

where *mp* and *mn* are the respective proton and neutron masses and *N* is the neutron number. Nuclear masses are experimentally measured accurately by mass spectrometers. Measurements by mass spectrometer, however, show that

$$\text{real nuclear mass} < Zmp + Nmn.$$

The difference in masses

$$Zmp + Nmn - \text{real nuclear mass} = \Delta m$$

is called the *mass defect.*

Stability of Nucleus and Binding Energy

$$\text{B.E per nucleon} = \frac{The\ number\ of\ nucleons\ it\ contains}{the\ bindng\ energy\ per\ nucleon\ it\ contains}$$

The Binding Energy per nucleon is plotted as a function of mass number *A* in Fig. 1.1. The curve rises steeply at first and then more gradually until it reaches a maximum of 8.79 *MeV* at $A = 56$, corresponding to the iron nucleus ${}_{26}Fe^{56}$. The curve then drops slowly to about 7.6 *MeV* at the highest mass numbers. Evidently, nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be split into lighter ones or if light nuclei can somehow be joined to form heavier ones.

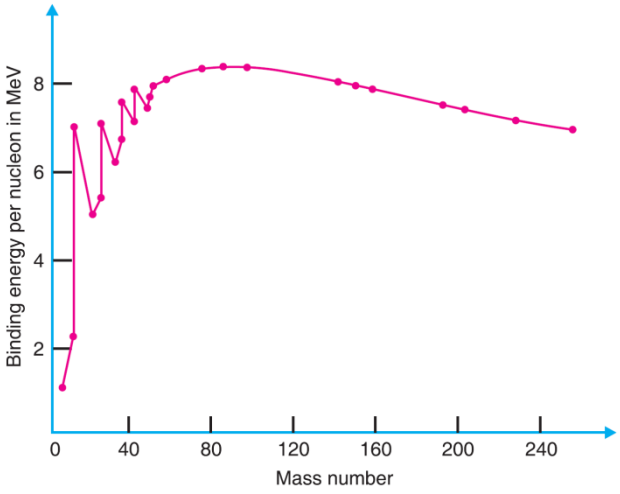


Fig 1.1

The former process is known as *nuclear fission* and the latter as *nuclear fusion.* Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

Packing fraction.

The ratio between the mass defect (Δm) and the mass number (A) is called

the packing fraction (f);

$f = Dm/A.$

Packing fraction means the mass defect per nucleon. Since atomic masses are measured relativeto $C^{-}12$, the packing fraction for this isotope is zero. Packing fraction is a measure of the comparative stability of the atom.

Packing fraction is defined as,

$$\text{Packing fraction} = \frac{Isotopic\ mass - Mass\ Number}{Mass\ Number} \text{ X } 10^4$$

Packing fraction may have a *negative* or a *positive* sign. If packing fraction is negative, the isotopic mass is less than the mass number. In such cases, some mass gets transformed into energy in the formation of that nucleus, in accordance with Einstein's equation $E = mc2$.

Such nuclei, therefore, are more stable. A positive packing fraction would imply a tendency towards instability. But this is not quite correct, especially for elements of low atomic masses.

A plot of packing fraction against the corresponding mass numbers of the various elements is shown in Fig. 1.2. It is seen that helium, carbon and oxygen atoms of mass numbers 4, 12 and 16 respectively, do not fall on this curve. Their paking fractions have small values. These elements are, therefore stable.

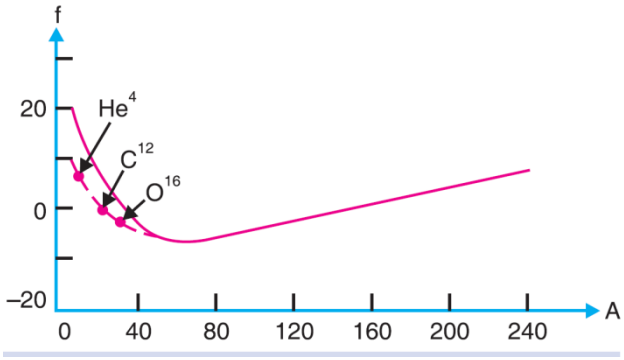


Fig 1.2

The transition elements, with mass numbers in the neighbourhood of 45, have lowest packing fractions with a negative sign, which indicates their high stability. The packing fraction beyond mass number 200 becomes positive and increases with increase in mass number. This indicates increasing instability of these elements. Elements with mass numbers beyond 230 are radioactive and undergo disintegration spontaneously.

Nuclear size.

Rutherford's work on the scattering of α-particles showed that the mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m. Thus the nucleus is about 10000 times smaller in radius than the atom. The empirical formula for the nuclear radius is

$$R = r_o A^{1/3}$$

where A is the mass number and r_o = 1.3 × 10–15 m = 1.3 *fm*. Nuclei are so small that the fermi (*fm*) is an appropriate unit of length. 1*fm* = 10–15 m. From this formula we find that the radius of the ${}_6C^{12}$ nucleus is $R \approx (1.3)\ (12)1/3 = 3fm$. Similarly, the radius of the ${}_{47}Ag^{107}$nucleus is 6.2 *fm* and that of the ${}_{92}U^{238}$ nucleus is 8.1 *fm*. The nuclear radius may be estimated from the scattering of neutrons and electrons by the nucleus, or by analysing the effect of the finite size of the nucleus on nuclear and atomic binding energies. The Fast neutrons of about 100 MeV energy, whose wavelength is small compared to the size of the nucleus, are scattered by nuclear targets. The fraction of neutrons scattered at various angles can be used to deduce the nuclear size. The results of these experiments indicate that the radius of a nucleus is given by $R \approx r_0 A^{1/3}$ where $r_0 \approx 1.3 - 1.4$ *fm*. The scattering can be done with proton beams as well. In this case, however, the effects due to Coulomb interaction have to be separated out. The observations are in agreement with the equation $R \approx r_0 A^{1/3}$ with $r_0 \approx 1.3 - 1.4$ *fm*.

The scattering of fast electrons of energy as high as 104 MeV, with a wavelength of about 0.1 *fm*, has the advantage that it can directly measure the charge density inside a nucleus. The results of the experiment are in agreement with the equation $R \approx r_0 A^{1/3}$ but with a somewhat smaller value of $r_0 \approx 1.2$ *fm*. The slight difference in the value of r_o may be ascribed to the fact that the electron scattering measures the charge density whereas the neutron and proton scattering experiments measure the region of large nuclear potential, which may be expected to be somewhat larger than the size of the nucleus.

Example. *The radius of* Ho^{165} *is* 7.731 *fermi. Deduce the radius of He*4.

Nuclear spin.

Electrons and protons have a spin of 1/2. Thus nuclei with an even number of protons and electrons should have integral spins, while those with an odd number of protons and electrons should have half-integral spins. Let us consider deuteron as an example. Deuteron nucleus has 3 particles (two protons and one electron). Hence the nuclear spin of deuteron should be 1/2 or 3/2. But experiment shows that the spin of the deuteron is 1. Thus the experimental result is in contradiction to the hypothesis.

Nuclear Energy Level Scheme and Explanation of Magic Numbers.

To account for the observed magic numbers, Mayer and Jensen postulated a strong *nuclear spin-orbit interaction*. The magnitude of the spin-orbit interaction is such that the consequent splitting of energy levels into sublevels is many times larger than the analogous splitting of atomic energy levels. The nuclear spin-orbit splitting of a single-nucleon energy level is assumed to be *large* and also *inverted* (Fig. 1.3). We ascribe this behaviour to a nuclear interaction of the form

$$V_{SL}\,(\text{nucleus}) = -\frac{a_{SL}^2}{r}\frac{dV}{dr}\mathbf{S}.\mathbf{L}.$$

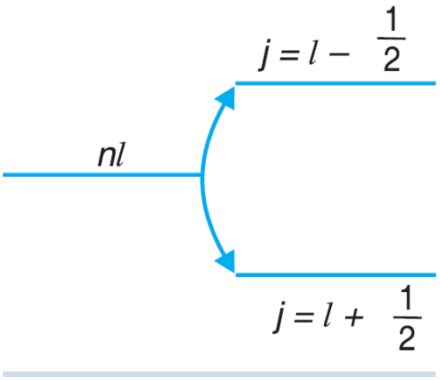


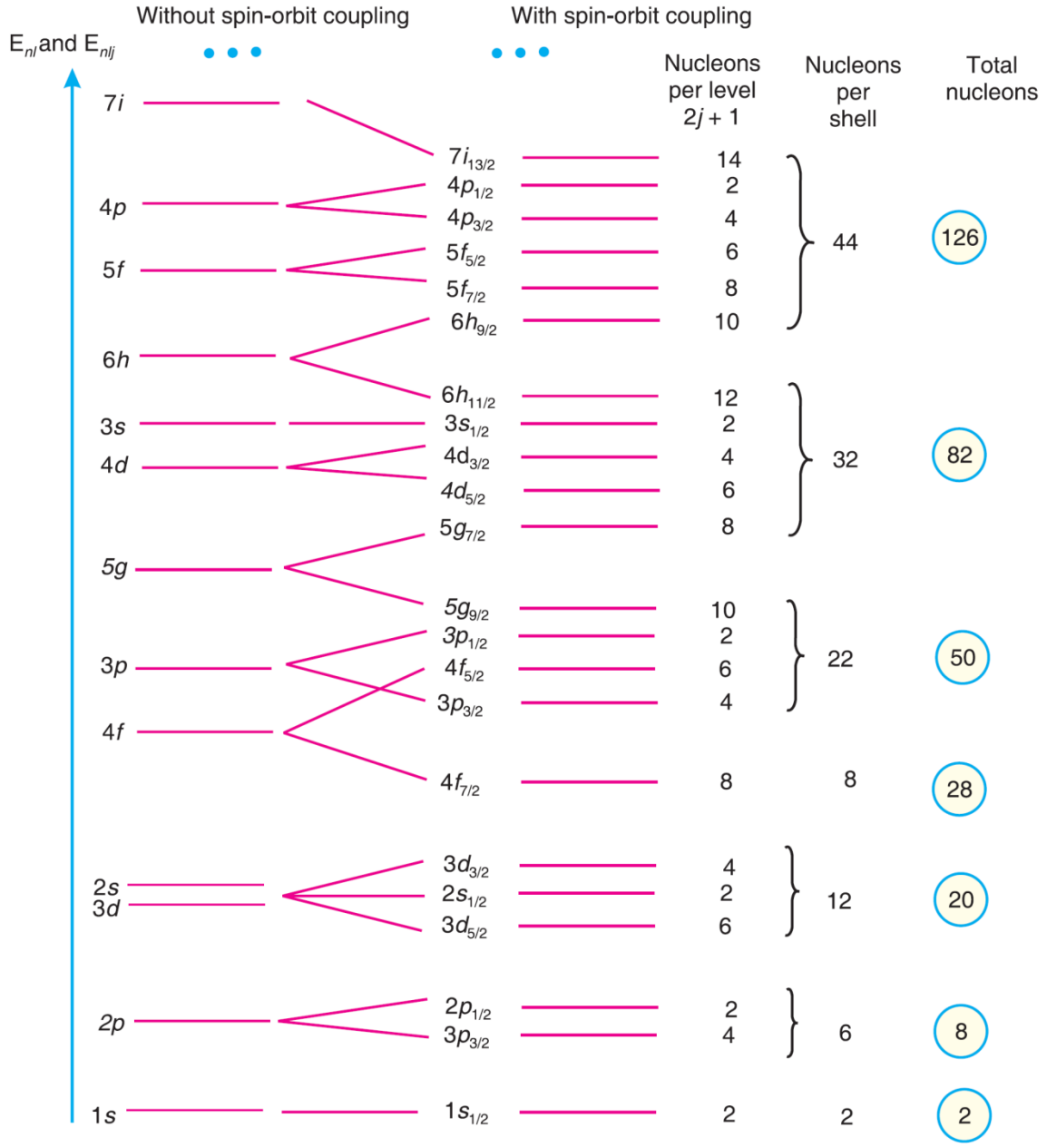
Fig 1.3

The minus sign accomplishes the required inversion of the split levels. The constant *a*2 *SL* produces the desired amount of energy splitting. The central-field function *V* (*r*) appears along with the orbital and spin angular momenta of the nucleon. The exact form of the potential-energy function is not critical, provided that it more or less resembles a square well. The shell theory assumes that *LS* coupling holds only for the very lightest nuclei, in which the *l* values are necessarily small in their normal configurations.

In this scheme, the intrinsic spin angular momenta *Si* of the particles concerned are coupled together into a total spin momentum **S**. The orbital angular momenta **L***i* are separately coupled together into a total orbital momentum **L**. Then **S** and **L** are coupled to form a total angular momentum **J** of magnitude $\sqrt{\mathbf{J}\,(\mathbf{J}+1)}\,\hbar$. After a transition region in which an intermediate coupling scheme holds, the heavier nuclei exhibit **jj coupling**. In this case, the **S***i* and **L***i* of each particle are first coupled to form a **J***i* for that particle of magnitude $\sqrt{\mathbf{J}\,(\mathbf{J}+1)}\,\hbar$. The various **J***i* then couple together to form the total angular momentum **J**. The *jj* coupling scheme holds for the great majority of nuclei.

Fig. 28.5 shows the nucleon energy levels according to the shell model. The levels are designated by a prefix equal to the total quantum number *n*, a letter that indicates *l* for each particle in that level, and a subscript equal to *j*. The spin-orbit interaction splits each state of given *j* into $2j + 1$ substates. The accumulated population of nucleons corresponds to a

magic number at every one of the larger energy gaps. Hence shells are filled when there are 2, 8, 20, 28, 50, 82 and 126 neutrons or protons in a nucleus.



Magnetic moment.

Protons and electrons are endowed with magnetic properties. The magnetic moment of an electron is about one thousand times that of a proton. If electrons exist inside the nucleus, the magnetic moment of electrons will have a dominating influence and so nuclear magnetic moments ought to be of the same order of magnitude as that of the electron. However, the observed magnetic moments of nuclei are comparable with that of the proton. This experimental fact goes against the electrons existing inside the nucleus. Due to these reasons, it is concluded that electrons cannot exist in the nucleus. Hence the proton-electron hypothesis regarding the constitution of the nucleus has been given up.

Proton-Neutron hypothesis.

After the discovery of neutron by Chadwick in 1932, proton neutron theory has been gaining support. According to this, a nucleus of atomic number *Z* and mass number *A* consists of *Z* protons and *A*–*Z* neutrons. The number of electrons in the extra nuclear space is also *Z*, so that the atom as a whole is neutral.

Parity of Nuclei.

The total spin of a nucleus consists of the sum of orbital angular momentum of nucleons and the sum of their spins. The orbital angular momentum $\sum_n ln$ actually defines the parity of the nuclei. The angular momentum eigen function can be expressed as a Spherical Harmonics *Ym* which is an even function for even *ln* and an odd function for an odd *ln*. This shows that $\sum_n ln$ is either even or odd for nuclei. According to this, the nuclear wave functions are said to have either an even parity or odd parity.

For positive or even parity, ψ (x, y, z) = ψ (– x, – y, – z).

For negative or odd parity, ψ (x, y, z) = – ψ (– x, – y, – z).

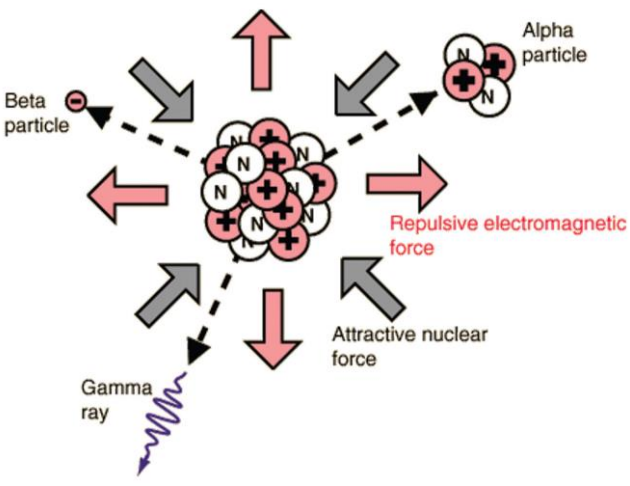
If $\sum_n ln$ is even, the parity of nuclide is positive and if $\sum_n ln$ is odd, the parity is negative. As an example, the Deuteron nucleus contains a neutron and a proton in *S*-state with l = 0 and the parity of deuteron is positive. Nuclei of various atoms in a ground state have a definite parity which is either positive or negative. When the nuclei are in an excited state, their parities are not always the same as in the ground state. Parity in nuclear transformations is conservable quantity but is not conserved in weak interactions like Beta decay. It is conserved in nuclear reactions and gamma decay.

Nuclear Forces

Since stable nuclei exist, it follows that there must be certain forces acting between their nucleons that bind them into the nucleus. These are called nuclear forces. The nuclear force must be strongly attractive, in order to overcome the electrostatic repulsion between protons. Only three kinds of attractive forces can be conceived in the nucleus, viz., neutron-neutron (n-n), neutron proton (n-p) and proton-proton (p-p) interactions. Yukawa attributed the following characteristics to the nuclear forces.

(1) Nuclear forces are effective only at short ranges.

Nuclear forces are appreciable only when the distance between nucleons is of the order of 10⁻15 m or less. The force vanishes for all practical purposes at distances greater than a few times 10⁻15 m. These distances are called the action radii or range of the nuclear forces. In the up-to-date version of the exchange theory of nuclear forces, it is supposed that interaction between nucleons is accomplished by the exchange of π-mesons. The exchange version of nuclear forces explains their short range action.



Let m be the rest mass of the π-meson. The rest energy of the π-meson = ΔE =

$mc2$. According to Heisenberg's uncertainty principle, the time required for nucleons to exchange π-mesons cannot exceed Δt, for which $\Delta E \Delta t \geq h/2\pi$. The distance that a π-meson can travel away from a nucleon in the nucleus during the time Δt, even at a velocity ≈ c, is $R_0 \approx (h/2\pi)/mc \approx 1.2 \times 10^{-15}$ m.

This approximately coincides with the value of the nuclear radius and is of the order of magnitude of the nuclear force range.

(2) Nuclear forces are charge independent. The nuclear forces acting between two protons, or between two neutrons, or between a proton and a neutron, are the same. It follows that nuclear forces are of a non-electric nature.

(3) Nuclear forces are the strongest known forces in nature.

(4) Nuclear forces have saturation property. Nuclear forces are limited in range. As a result, each nucleon interacts with only a limited number of nucleons nearest to it. This effect is referred to as the *saturation* of nuclear forces.

Models of Nuclear Structure

The precise nature of the forces acting in the nucleus is unknown. Hence, nuclear models are resorted to for investigation and theoretical prediction of its properties. Such models may be based on (*i*) the extrinsic analogy between the properties of atomic nuclei and those of a liquid drop (*ii*) the electron shell of an atom etc. The corresponding models are called the liquid-drop model, shell model, etc.

The Liquid Drop Model

In the liquid-drop model, the forces acting in the nucleus are assumed to be analogical to the molecular forces in a droplet of some liquid. This model was proposed by Neils Bohr who observed that there are certain marked similarities between an atomic nucleus and a liquid drop. The similarities between the nucleus and a liquid drop are the following:

(i) The nucleus is supposed to be spherical in shape in the stable state, just as a liquid drop is spherical due to the symmetrical surface tension forces.

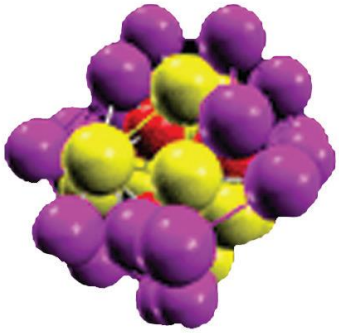
(ii) The force of surface tension acts on the surface of the liquid drop. Similarly, there is a potential barrier at the surface of the nucleus.

(iii) The density of a liquid-drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

(iv) The intermolecular forces in a liquid are short range forces. The molecules in a liquid drop interact only with their immediate neighbours. Similarly, the nuclear forces are short range forces. Nucleons in the nucleus also interact only with their immediate neighbours. This leads to the saturation in the nuclear forces and a constant binding energy per nucleon.

(v) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

(vi) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.



The Liquid Drop Model

Semi-empirical mass formula.

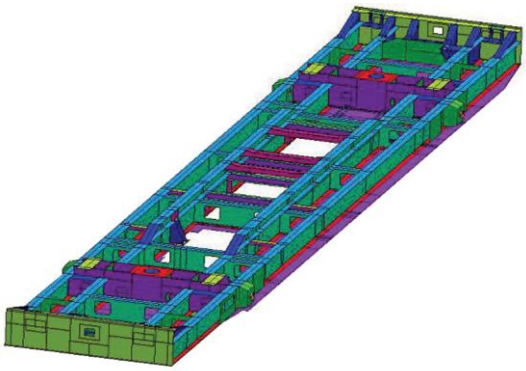
The liquid-drop model can be used to obtain an expression for the binding energy of the nucleus. Weizacker proposed the semi-empirical nuclear binding energy formula for a nucleus of mass number *A*, containing *Z* protons and *N* neutrons. It is written as

$$\text{B.E} = \text{Aa} - \text{b}A^{2/3} - \frac{cZ(Z-1)}{A^{1/3}} - \frac{d(N-Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

where *a*, *b*, *c*, *d* and δ are constants.

The Shell Model

The shell model of the nucleus assumes that the energy structure (energy levels of the nucleons) of the nucleus is similar to that of an electron shell in an atom. According to this model, the protons and neutrons are grouped in shells in the nucleus, similar to extra-nuclear electrons in various shells outside the nucleus. The shells are regarded as "filled" when they contain a specific number of protons or neutrons or both. The number of nucleons in each shell is limited by the *Pauli exclusion principle*. The shell model is sometimes referred to as the *independent particle model* because it assumes that each nucleon moves independently of all the other nucleons and is acted on by an average nuclear field produced by the action of all the other nucleons



Evidence for shell model.

It is known that a nucleus is stable if it has a certain definite number of either protons or neutrons. These numbers are known as *magic numbers*. The magic numbers are 2, 8, 20, 50, 82 and 126. Thus nuclei containing 2, 8, 20, 50, 82 and 126 nucleons of the same kind form some sort of closed nuclear shell structures. The main points in favour of this inference are:

(*i*) The inert gases with closed electron shells exhibit a high degree of chemical stability. Similarly, nuclides whose nuclei contain a magic number of nucleons of the same kind

exhibit more than average stability.

(*ii*) Isotopes of elements having an isotopic abundance greater than 60% belong to the magic number category.

(*iii*) Tin (50Sn) has ten stable isotopes, while calcium (20Ca40) has six stable isotopes. So elements with Z = 50, 20 are more than usually stable.

(*iv*) The three main radioactive series (viz., the uranium series, actinium series and thorium series) decay to 82*Pb*208 with Z = 82 and N = 126. Thus lead 82*Pb*208 is the most stable isotope. This again shows that the numbers 82 and 126 indicate stability.

(*v*) It has been found that nuclei having a number of neutrons equal to the magic number, cannot capture a neutron because the shells are closed and they cannot contain an extra neutron.

(*vi*) It is found that some isotopes are spontaneous neutron emitters when excited above the nucleon binding energy by a preceding β-decay. These are 8*O*17, 36*Kr*87 and 54*Xe*137 for which N = 9, 51 and 83 which can be written as 8 + 1, 50 +1, and 82 + 1. If we interpret this loosely bound neutron, as a valency neutron, the neutron numbers 8, 50, 82 represent greater stability than other neutron numbers.

It is apparant from the above conclusions that nuclear behaviour is often determined by the excess or deficiency of nucleons with respect to closed shells of nucleons corresponding to the magic numbers. It was, therefore, suggested that nucleons revolve inside the nucleus just as electrons revolve outside in specific permitted orbits. The protons and neutrons move in two separate systems of orbits round the centre of mass of all the nucleons. The extra-nuclear electrons revolve in the Coulomb field of a relatively distant heavy nucleus. But the nucleons move in orbits around a common centre of gravity of all the constituents of the nucleus. Each nucleon shell has a specific maximum capacity. When the shells are filled to capacity, they give rise to particular numbers (the magic numbers) characteristic of unusual stability.

The shell model is able to account for several nuclear phenomena in addition to magic numbers.

(*i*) It is observed that even-even nuclei are, in general, more stable than odd-odd nuclei. This is obvious from the shell model. According to Pauli's principle, a single energy sublevel can have a maximum of two nucleons (one with spin up and other with spin down). Therefore, in an even-even nucleus only *completed* sublevels are present which means greater stability. On the other hand, an odd-odd nucleus contains incomplete sublevels for both kinds of nucleon which means lesser stability.

(*ii*) The shell model is able to predict the total angular momenta of nuclei. In even-even nuclei, all the protons and neutrons should pair off so as to cancel out one another's spin and orbital angular momenta. Thus even-even nuclei ought to have zero nuclear angular momenta, as observed. In even odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momentum of the rest of nucleus for a half-integral total angular momentum. Odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta. Both these predictions are experimentally confirmed.

UNIT – II RADIO ACTIVITY

Radio active decay law:

The half-life period of a radioactive substance is defined as the time required for one-half of the radioactive substance to disintegrate**.**

Value of half-life period

We know the relation N = N0 e- λ t.

If T1/2 be the half-life period, then at t = $T_{1/2}$, N = $N_0/2$.

$$\therefore \ N_0/2 = \log_e 2 \text{ or } e^{\lambda T}1/2 = 2$$

$$\text{or } \lambda T_{1/2} = \log_e 2 \text{ or } T1/2 = \frac{\log e2}{\lambda}$$

$$\therefore T_{1/2} = \frac{\log e2}{\lambda} \text{ or } \frac{0.6931}{\lambda}$$

Example 1. The half-value period of radium is 1590 years. In how many years will one gram of pure element (a) lose one centigram, and (b) be reduced to one centigram ?

$$\therefore \text{ Radioactive constant } = \lambda = \frac{0.6931}{T_{\frac{1}{2}}} = \frac{0.6931}{1590}.$$

(*a*) Let *t* be the time in which one gram of radium loses one centigram (0.01g).

$\therefore$ Radium left behind = 1 – 0.01 = 0.99 gram.

Now, $N = N_0 e^{-\lambda t}$ or $\log_e N = \log_e N_0 - \lambda t$

or $$\lambda t = \log_e \left(\frac{N_0}{N}\right)$$

$\therefore$ $$t = \frac{1}{\lambda}\log_e\left(\frac{N_0}{N}\right) = \frac{1590}{0.6931}\log_e\left(\frac{1}{0.99}\right)$$

$$= 23.25 \text{ years.}$$

(*b*) Here, $N = 0.01$ gram; $N_0 = 1$ gram; $t = ?$

$$t = \frac{1}{\lambda}\log_e\left(\frac{N_0}{N}\right) = \frac{1590}{0.6931}\log_e\left(\frac{1}{0.01}\right) = 10560 \text{ years.}$$

Example 2. 1 gram of radium is reduced by 2.1 mg in 5 years by α -decay. Calculate the halflife period of radium

Sol. Initial mass of radium = 1 gram

Mass of radium left behind after 5 years = 1 – 0.0021 = 0.9979 g.

We have, $N = N_0\, e^{-\lambda t}$; Here, $\frac{N}{N_0} = \frac{0.9979}{1}$; $t = 5$ years.

$$\therefore \quad 0.9979 = e^{-5\lambda} \text{ or } e^{5\lambda} = \frac{1}{0.9979}$$

$$\text{or} \quad 5\lambda = \log_e\left(\frac{1}{0.9979}\right)$$

$$\therefore \quad \lambda = \frac{1}{5}\log_e\left(\frac{1}{0.9979}\right) = \frac{2.3026}{5}\log_{10}\left(\frac{1}{0.9979}\right)$$

$$= 41.45 \times 10^{-5} \text{ per year.}$$

$$T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{41.45 \times 10^{-5}} = 1672 \text{ years.}$$

Example 3. Calculate the time required for 10% of a sample of thorium to disintegrate. Assume the half-life of thorium to be 1.4 × 1010 years.

Sol. We have, $N = N_0\, e^{-\lambda t}$. Here $N = 0.9\, N_0$.

$$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{1.4 \times 10^{10}} a^{-1}.$$

$$\therefore \quad 0.9\, N_0 = N_0\, e^{-\lambda t} \quad \text{or} \quad \lambda t = \log_e\left(\frac{1}{0.9}\right) \text{or } t = \frac{1}{\lambda}\log_e\left(\frac{1}{0.9}\right)$$

$$t = \frac{1.4 \times 10^{10}}{0.6931} \times \log_e 1.111 = \frac{1.4 \times 10^{10}}{0.6931} \times 2.302 \times \log_{10} 1.111$$

$$= 2.1 \times 10^9 \text{ years.}$$

The Mean life

It is not possible to predict which atom of a radioactive substance will disintegrate at any instant. The atom which disintegrates first has zero life and that disintegrates last has infinite life. Thus the life of every atom is different and the actual lives of the various atoms range from zero to infinity.

$$\therefore \text{ The mean-life of a radioactive element} = \frac{\text{sum of the lives of all the atoms}}{\text{Total number of atoms}}$$

Definition. The mean-life of a radioelement is defined as the ratio of the total life time of all the radioactive atoms to the total number of such atoms in it.

Value of mean-life. Let N0 be the total number of radioactive atoms in the beginning. Let N be the number of atoms of that element after time t. Then $N = N_0\, e^{-\lambda t}$. Let dN be the number of atoms disintegrating between time t and $t + \delta t$. These dN atoms have had a life between t and $(t + \delta t)$. Since δt is very small, each of these atoms had a life of t.

∴ total life of dN atoms = (dN)t.

The possible life of any of the total number N0 radioactive atoms varies from 0 to ∞.

∴ Total life time of all N_0 atoms = $\int_0^\infty t\, dN$

Now, mean life = T = $\frac{\text{total life–time}}{\text{total number of atoms}}$

$$= \frac{\int_0^\infty t\,dN}{N_0}$$

Now, $N = N_0\, e^{-\lambda t}$

$\therefore$ $\dfrac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$

or $dN = -\lambda\, N_0\, e^{-\lambda t}\, dt$

$dN = \lambda N_0\, e^{-\lambda t}\, dt$

(Leaving the negative sign which merely indicates the decrease in the number of atoms with time).

Hence, $\bar{T} = \dfrac{\int_0^\infty t\lambda N_0\, e^{-\lambda t}\, dt}{N_0} = \lambda \int_0^\infty t\, e^{-\lambda t}\, dt$

Integrating by parts,

$$\bar{T} = \lambda\left[\frac{te^{-\lambda t}}{-\lambda} - \int \frac{e^{-\lambda t}\, dt}{-\lambda}\right]_0^\infty$$

$$= \lambda\left[\frac{te^{-\lambda t}}{-\lambda} - \frac{e^{-\lambda t}}{-\lambda^2}\right]_0^\infty = \lambda\left(\frac{1}{\lambda^2}\right) = \frac{1}{\lambda}$$

$\therefore$ $\bar{T} = \dfrac{1}{\lambda}$.

Thus the mean life (T) of a radioactive substance is the reciprocal of the decay constant (λ).

Example 1. 1 gram of a radioactive substance disintegrates at the rate of 3.7×10^{10} disintegrations per second. The atomic weight of the substance is 226. Calculate its mean life.

SOL. Number of atoms disintegrated in one second = 3.7×10^{10}

The mass of the substance disintegrated in one second = $\dfrac{(3.7 \times 10^{10}) \times 226}{6.02 \times 10^{26}}$

$= 1.389 \times 10^{-14}$ kg

Here, $N = 1$ g $= 10^{-3}$ kg; $-dN/dt = 1.389 \times 10^{-14}$ kg

We have, $-dN/dt = \lambda N$

or $\lambda = \dfrac{-dN/dt}{N} = \dfrac{1.389 \times 10^{-14}}{10^{-3}} = 1.389 \times 10^{-11}\, s^{-1}$

$\therefore$ Mean life $\bar{T} = \dfrac{1}{\lambda} = \dfrac{1}{1.389 \times 10^{-11}}\, s = 2282\ years$.

Example 2. The disintegration constant λ of a radioactive element is 0.00231 per day. Calculate its half-life and average life.

SOL. Here, $\lambda = 0.00231$ per day.

(*i*) Half-life period $= T_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{0.693}{0.00231} = 300$ days

(*ii*) Average life period $= \overline{T} = \dfrac{1}{\lambda} = \dfrac{1}{0.00231} = 432.9$ days.

Activity or Strength of Radio.

The *activity* of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity R is given by $R = -dN/dt$. The S.I. unit of activity is named after Henri Becquerel.

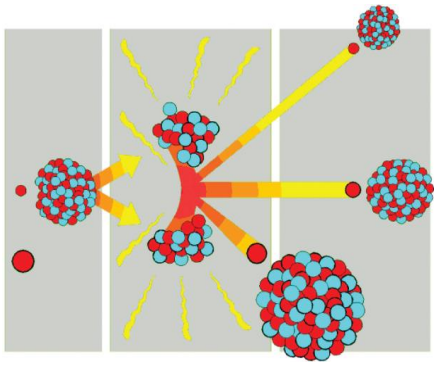
1 becquerel = 1 Bq = 1 event/s.

1 MBq = 106 Bq and 1 GBq = 109 Bq.

Chain Reaction

A chain reaction is a self-propagating process in which number of neutrons goes on multiplying rapidly almost in geometrical progression during fission till whole of fissile material is disintegrated.

Example : Suppose a single neutron causing fission in a uranium nucleus produces 3 prompt neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing 9 neutrons. These nine neutrons in turn may cause fission in nine uranium nuclei producing 27 neutrons and so on. The number of neutrons produced in n such generations is 3^n. The ratio of secondary neutrons produced to the original neutrons is called the *multiplication factor* (k).



Chain Reaction

Consider 1 kg of ${}_{92}U^{235}$ which contains $6.023 \times 10^{26}/235$ or about 25×10^{23} atoms. Suppose a stray neutron causes fission in a uranium nucleus. Each fission will release on the average 2.5 neutrons. The velocity of a neutron among the uranium atoms is such that a fission capture of a thermal neutron by the ${}_{92}U^{235}$ nuclei takes place in about 10^{-8} *s*. Each of these fissions, in turn, will release 2.5 neutrons. Let us assume that all these neutrons are available for inducing further fission reactions. Let n be the number of stages of fission captures required to disrupt the entire mass of 1 kg of ${}_{92}U^{235}$. Then

$$(2.5)n = 25 \times 1023 \text{ or } n \approx 60.$$

The time required for 60 fissions to take place = 60×10^{-8} *s* = 0.6 μ *s*.

Since each fission releases about 200 MeV of energy, this means that a total of $200 \times 25 \times 10^{23} = 5 \times 10^{26}$ MeV of energy is released in 0.6 μ s. The release of this tremendous amount of energy in such a short time interval leads to a violent explosion. This results in powerful air blasts and high temperature of the order of 10^7 *K* or more, besides intense radioactivity. The self-propagating process described here is called a *chain reaction.*

Two types of chain reaction are possible. In one, the chain reaction is first accelerated so that the neutrons are built up to a certain level and there after the number of fission producing neutrons is kept constant. This is *controlled chain reaction*. Such a controlled chain reaction is used in nuclear reactors. In the other type of chain reaction, the number of neutrons is allowed to multiply indefinitely and the entire energy is released all at once. This type of reaction takes place in atom bombs.

Multiplication factor *(k). The ratio of secondary neutrons produced to the original neutrons is called the multiplication factor. It is defined as*

$$K = \frac{Number\ of\ neutrons\ in\ any\ one\ generation}{Number\ of\ neutrons\ in\ the\ preceding\ generation}$$

The fission chain reaction will be "*critical*" or steady when $k = 1$, it will be building up or "*Supercritical* " when $k > 1$ and it will be dying down or "*subcritical* " when $k < 1$.

Critical size for maintenance of chain reaction.

Consider a system consisting of uranium (as fissile material) and a moderator. Even though each neutron that produces fission ejects 2.5 neutrons on an average, all of them are not available for further fission. The maintenance of the chain reaction depends upon a favourable balance of neutrons among the three processes given below :

(1) The fission of uranium nuclei which produces more neutrons than the number of neutrons used for inducing fission.

(2) Non-fission processes, including the radiative capture of neutrons by the uranium and the parasitic capture by the different substances in the system and by impurities.

(3) Escape or leakage of neutrons through the surface of the system. If the loss of neutrons due to the last two causes is less than the surplus of neutrons produced in the first, a chain reaction takes places. Otherwise it cannot take place.

The escape of neutrons takes place from the surface of the reacting body and fission occurs throughout its *volume.*

∴ Escape rate varies as r^2 and production rate varies as r^3.

$$\therefore \frac{Escape\ rate}{production\ rate} \propto \frac{1}{r}$$

The larger the size of the body, the smaller is the escape rate. Thus it is clear that by increasing the volume of the system, the loss of neutrons by escape from the system is reduced. The greater the size of the system, the lesser is the probability of the escape of neutrons. In this case, the production of neutrons will be more than the loss due to other causes and a chain reaction can be maintained. Thus there is a critical size for the system. *Critical size of a system containing fissile material is defined as the minimum size for which the number of neutrons produced in the fission process just balance those lost by leakage and non-fission capture*. The mass of the fissionable material at this size is called the *critical mass*. If the size is less than the critical size, a chain reaction is not possible.

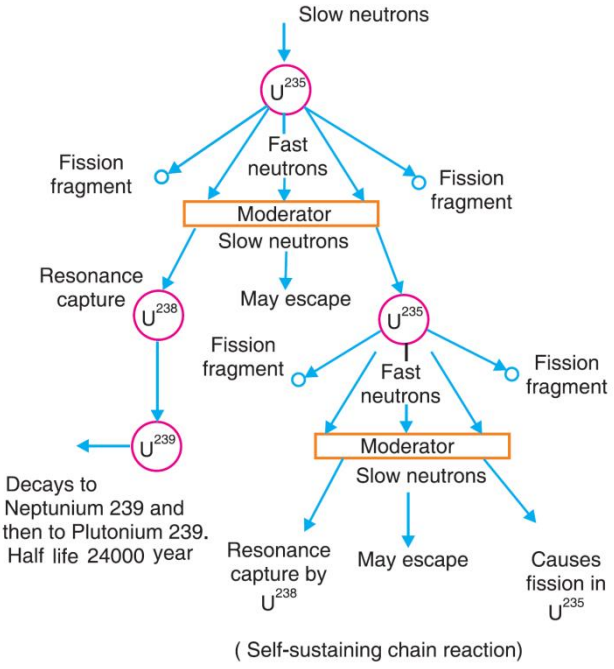


Fig 2.1

Radioactive equilibrium :

Case (*i*). Secular or permanent equilibrium. Suppose $T1 >> T2$ with $T1 \approx \infty$ and $T2 = 0$. (*i.e.*, the half-life of *A* is very much longer than that of *B*).

Then, $\lambda_1 << \lambda_2$ and $\lambda_1 \approx 0$. In equation (2), $e^{-\lambda_1 t} \to 1$

$$\therefore \quad N_2 = \frac{N_0 \lambda_1}{\lambda_2}(1 - e^{-\lambda_2 t}) \qquad ...(3)$$

After an appreciably long time, $e^{-\lambda_2 t}$ becomes negligible and

$$N_1 \approx N_0.$$

$$\therefore \quad N_1 \lambda_1 = N_2 \lambda_2 \qquad ...(4)$$

This relation shows that at equilibrium, the rate of decay of any radioactive product is just equal to its rate of production from the previous member of the chain. Now the daughter is said to be in *secular* or *permanent equilibrium* with the parent. An example of this is provided by the decay of radium into radon according to the equation.

$$Ra^{226} \to Rn^{222} + \alpha .$$

The half-life period of *Ra* (1590 years) is very large compared with that of *Rn* (3.8 days). Fig. 2.2 shows the experimental decay and recovery curves. After a time, long enough compared with its mean life, *Rn* is in permanent equilibrium with *Ra*.

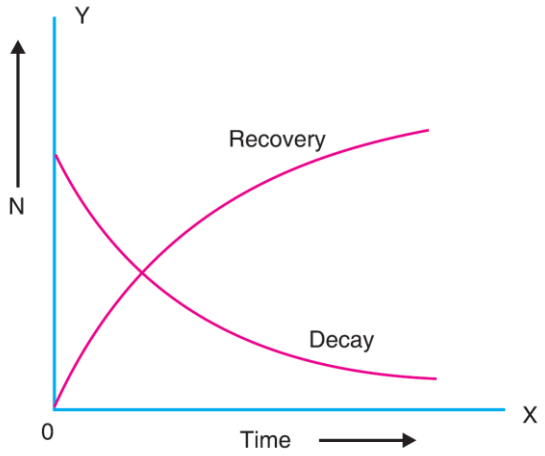


Fig 2.2

Its amount becomes constant as shown by the sum of the ordinates of the two curves which becomes constant beyond a certain value of time indicated by the point of intersection.

Case (*ii*). Transient equilibrium. Suppose $\lambda 1 << \lambda 2$ but $\lambda 1 \neq 0$ *i.e.*, the decay constant $\lambda 1$ which is very small compared with $\lambda 2$, is not small enough to be ignored. In this case, after a sufficiently long time, $e^{-\lambda 2 t}$ becomes negligible compared with $e^{-\lambda 1 t}$. Hence

$$N_2 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} = \frac{\lambda_1 N_1}{\lambda_2 - \lambda_1}$$

or

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \qquad ...(5)$$

In this case, both *A* and *B* decay while the ratio *N*2 / *N*1 remains constant.

Radioactive Dating : The Age of the Earth

The age of the earth is estimated from the relative abundance of the two isotopes of uranium, U^{238} and U^{235}. The half-periods of U^{238} and U^{235} are 4.5 × 109 years and 7 × 108 years respectively. Assume that at the beginning when the earth was formed the proportions of the two isotopes were equal. The present relative abundance of U^{238} to U^{235} in natural uranium is 99.3% to 0.7%.

$$\therefore \quad \frac{N_1}{N_2} = \frac{99.3}{0.7} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)t}$$

where $$\lambda_1 = \frac{0.6931}{4.5 \times 10^9} \text{ and } \lambda_2 = \frac{0.6931}{7 \times 10^8}$$

$$\therefore \quad \log_e \left(\frac{99.3}{0.7} \right) = (\lambda_2 - \lambda_1)t$$

$$\therefore \quad t = \frac{1}{\lambda_2 - \lambda_1} \log_e \left[\frac{99.3}{0.7} \right]$$

$$= \frac{1}{\left(\frac{0.6931}{7 \times 10^8} \right) - \left(\frac{0.6931}{4.5 \times 10^9} \right)} \log_e \left(\frac{99.3}{0.7} \right)$$

$$= 5.93 \times 10^9 \text{ years.}$$

This value agrees nearly with that given by astronomical evidence for the age of the universe.

Dating by radioactive decay. The decay of radioactive elements is independent of the physical and chemical conditions imposed on them. Although the decay of an individual particle from a given nucleus is a random process, the gross decay of the many nuclei in a given sample provides a very convenient way of measuring times.

In 1913 Joly and Rutherford suggested that if igneous rock, formed as a result of a prehistoric volcanic eruption, contanied a small amount of uranium it would steadily decay, leaving less uranium and depositing more stable Pb-206. By measuring the ratio of uranium to lead in rock samples, a rather exact time can be determined for the origin of the geological deposits. Uranium dating measures times of the order of millions of years.

Radioactive dating model

Since all plants use CO2 from the atmosphere for growth, a portion of the carbon in plants is radioactive C-14 and the plants are slightly radioactive. When a plant dies, no additional C-14 is taken in, and that within the plant body begins to decay without being replaced. Measurement of the relative amounts of C-14 and C-12 in an organic archeological sample provides a sensitive method of dating.

Example. A carbon specimen found in a cave contained 1/8 as much C14 as an equal amount of carbon in living matter. Calculate the approximate age of the specimen. Half-life period of C14 is 5568 years.

SOL. Here, $T_{1/2} = 5568 \text{ years}; T_{1/2} = \dfrac{0.6931}{\lambda}$

or $\lambda = \dfrac{0.6931}{1/2} \quad \dfrac{0.6931}{5568} \text{ year}^{-1}$

We have, $N = N_0 e^{-\lambda t}$ or $\dfrac{N}{N_0} = e^{-\lambda t}$ or $\dfrac{1}{8} = e^{-\lambda t}$

or $\lambda t = \log_e 8$

$\therefore \quad t = \dfrac{\log_e 8}{\lambda} = \dfrac{(2.3026 \log_{10} 8) \times 5568}{0.6931}$

$= 16710$ years.

Geiger and Nuttal experiment.

The α-particle source is in the form of a thin film on a small metal disc M (Fig.2.3). It is mounted at the centre of a spherical glass bulb B. The bulb is coated inside with silver and a high P.D. is applied between the silver film and the disc M. The saturation ionisation current in the bulb produced by the passage of the α-particles through the gas in the bulb is measured by an electrometer. The gas under study can be admitted at any desired pressure into the bulb. The saturation current for different gas pressures is measured. A graph is plotted between the ionisation current and the corresponding pressure P of the gas. We get curves of the type shown in Fig. 2.4.

The ionisation at first increases directly with the pressure. At a certain critical pressure Pc, the density of the gas is such that the particles are just stopped on the inner surface of the bulb, having produced the maximum ionisation possible, before reaching the bulb. If the pressure is increased beyond Pc, no rise in the current is observed, since the α-particles get absorbed in a smaller path and do not reach the wall. The radius of the bulb thus gives the range of the α-particles at the gas pressure Pc. As the range is inversely proportional to pressure, the range at normal pressure can be calculated.

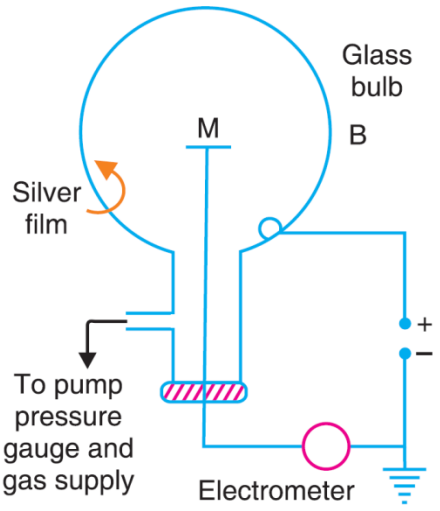


Fig 2.3

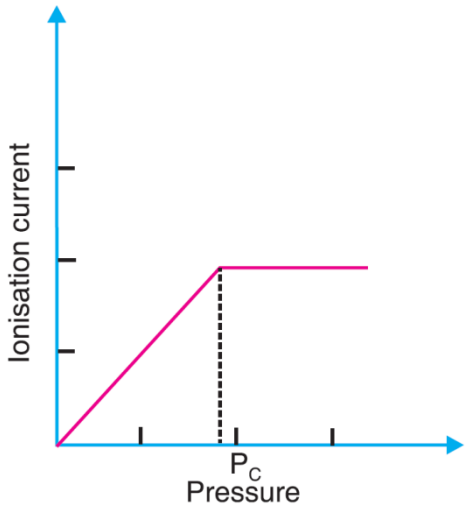


Fig 2.4

Geiger's law. Geiger studied the relation between the range (R) of an α-particle and its velocity of emission (v). He found that the range R is proportional to the cube of the velocity. R ∝ v3 or R = av3, where a is a constant.

Range-energy relation. The energy E of the emitted particle is directly proportional to the square of the velocity v. Hence Geiger's law can be expressed in the form R = bE3/2 where b is a constant.

Geiger-Nuttal law. The range R of an α-particle and the disintegration constant λ of the radioactive element that emits it are related as follows:

$$\log \lambda = A + B \log R.$$

This relation is called Geiger-Nuttal law. If log λ are plotted against log R for the different α-emitters in the three series, three nearly parallel straight lines are obtained, one for each series (Fig.2.5). In the relation log λ = A + B log R, the constant B is the same for all the series while A is different for the different series. According to this relation, when the disintegration constant is high, the range is also high. Since the range also depends on the energy, we conclude that radioactive substances of large decay constants emit high energy α-particles. This law is helpful in determining roughly the decay constants of radioactive substances of very short or very long lives. Experimentally measuring the ranges of α-particles of such radioelements, the respective decay constants can be obtained by extrapolation from the curves representing the Geiger-Nuttal relation.

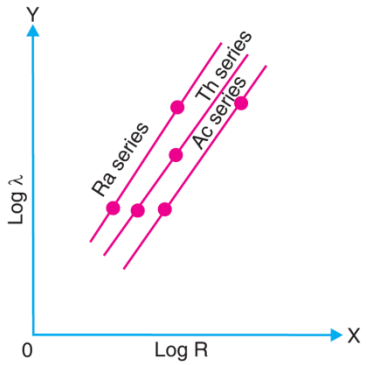


Fig 2.5

Theory of Alpha Decay

Only heavy nuclei with *A* > 200 undergo α-decay. The α-particles emitted from nuclei have a discrete energy spectrum and consist of several groups. Usually the most intensive is the group with α-particles of highest energy. The existence of several groups of α-particles is called the *fine structure of the* α*-spectrum.*

Before emission, the α-particle can be considered to be inside the nucleus. Coulomb's law is applicable when the α-particle is outside the nucleus. When the α-particle is inside the nucleus or very close to it, Coulomb's law does not hold good. For an α-particle of charge 2*e* and a nucleus of charge (Z–2) *e* separated by a distance *r*, the potential energy is given by

$$V_r = \frac{2e(Z-2)e}{4\pi\varepsilon_0 r} = \frac{2(Z-2)e^2}{4\pi\varepsilon_0 r}.$$

Here $(Z-2)$ is the atomic number of the daughter nucleus. An attractive potential, called the potential well, represents the position (Fig.2.6). When $r < r0$, the α-particle is within the potential well and it is bound by the nucleus. Here $r0$ is equal to the sum of the radii of the nucleus and the α-particle. According to wave mechanics, an α-particle can have different energy levels $E0$, $E1$, $E2$, etc., within the potential well. Only an α-particle at a level $E3$ or above can come out of the potential well. In the case of radioactive elements, the height of the potential barrier is about 9 MeV. But uranium emits α-particles of energy 4 MeV.

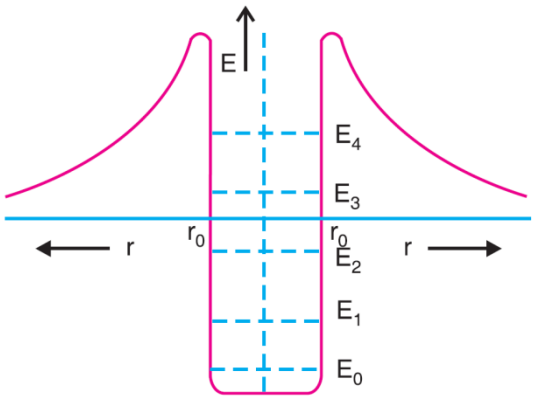


Fig 2.6

Classical mechanics cannot explain how a particle having an energy of 4 MeV can come out of a well having a potential barrier of 9 MeV.

The escape of an α-particle from a radioactive nucleus can be explained on the basis of wave mechanics and Schrodinger equation. According to it, it is possible for an α-particle to *leak* through the potential barrier, even though its K.E. is less than the potential energy of the height of potential barrier. This probability of leaking of an α-particle through the barrier is called *tunnel effect*. The α-particle within the nucleus must present itself again and again at the barrier surface until conditions are ripe for penetration or leakage

Gamow's Theory of Alpha Decay

Classical physics fails to explain α-decay. Quantum mechanics provides a successful explanation of the problem of α-decay. The basic notions of this theory are :

(1) An alpha particle may exist as an entity within a heavy nucleus.

(2) Alpha particle is in constant motion and bounces back and forth from the barrier walls.

In each collision with 'wall' there is a definite probability that the particle will leak through the potential barrier. Let ν be the frequency with which the α-particle collides with the walls in order to escape from the nucleus and P the probability of transmission in each collision. Then the decay probability per unit time (*i.e.*, disintegration constant λ) is given by

$$\lambda = \nu P \qquad ...(1)$$

Suppose that at any moment only one α-particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter.

$$\text{Then } v = v / 2R \qquad ....(2)$$

where v is the alpha particle velocity and R is the nuclear radius.
Since $V > T$ [where V = height of the potential barrier and T = K.E. of α-particle], according to classical physics $P = 0$ [Fig. 2.7]. But in quantum mechanics a moving particle is regarded as a wave, and the result is a small but definite value for P. The probability P can be calculated quantum mechanically using WKB perturbation theory.

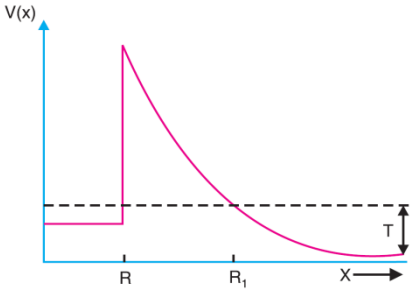


Fig 2.7

According to this theory,

$$\log_e P = \frac{-2}{(h/2\pi)} \int_R^{R_1} \sqrt{2m\{V(x) - T\}}\, dx \qquad ...(3)$$

where m = mass of the α-particle.

$$V(x) = \frac{2Ze^2}{4\pi\varepsilon_0 x}$$

is the eletrostatic P.E. of an alpha particle at a distance x from the centre of a nucleus of charge Ze (Ze is the charge of the daughter nucleus *i.e.*, nuclear charge minus the alpha particle charge of $2e$)

R = Nuclear radius

T = K.E. of α-particle such that $T < V(x)$.

The region from $x = R$ to $x = R1$ is called the thickness of the barrier.

We therefore have:

$$\log_e P = \frac{-2}{(h/2\pi)} \int_R^{R_1} \sqrt{2m\left(\frac{2Ze^2}{4\pi\varepsilon_0 x} - T\right)}\, dx \qquad ...(4)$$

When $x = R_1, T = V. \quad \therefore T = \dfrac{2Ze^2}{4\pi\varepsilon_0 R_1}$

$$\therefore \quad \log_e P = \frac{-2}{(h/2\pi)} (2mT)^{1/2} \int_R^{R_1} \left(\frac{R_1}{x} - 1\right)^{1/2} dx$$

$$\log_e P = \frac{-2}{(h/2\pi)} (2mT)^{1/2} R_1 \left[\cos^{-1}\left(\frac{R}{R_1}\right)^{1/2} - \left(\frac{R}{R_1}\right)^{1/2} \left(1 - \frac{R}{R_1}\right)^{1/2}\right] \qquad ...(5)$$

The width of the potential barrier is very large compared with the nuclear radius *i.e.*, $R_1 >> R$. Therefore

$$\cos^{-1}\left(\frac{R}{R_1}\right)^{1/2} \approx \frac{\pi}{2} - \left(\frac{R}{R_1}\right)^{1/2} \text{ and } \left(1 - \frac{R}{R_1}\right)^{1/2} \approx 1$$

Hence $$\log_e P = \frac{-2}{(h/2\pi)}(2mT)^{1/2} R_1 \left(\frac{\pi}{2} - 2\left(\frac{R}{R_1}\right)^{1/2}\right)$$

Substituting $R_1 = 2Ze^2 / 4\pi\varepsilon_0 T$, we have

$$\log_e P = \frac{4e}{(h/2\pi)}\left(\frac{m}{\pi\varepsilon_0}\right)^{1/2} Z^{1/2} R^{1/2} - \frac{e^2}{(h/2\pi)\varepsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} Z T^{-1/2} \quad ...(6)$$

or $$\log_e P = 2.97 Z^{1/2} R^{1/2} - 3.95 ZT^{-1/2}$$

where R is in fermi, and T is in *MeV*.

Since $$\lambda = \nu P, \log_e \lambda = \log_e \nu + \log_e P = \log_e\left(\frac{v}{2R}\right) + \log_e P$$

$$\therefore \quad \log_e \lambda = \log_e\left(\frac{v}{2R}\right) + 2.97 Z^{1/2} R^{1/2} - 3.95 Z T^{-1/2} \quad ...(7)$$

Changing the base of log (from e to 10),

$$\log_{10} \lambda = \log_{10}\left(\text{——}\right)\ 0.4343(2.97 Z^{1/2} R^{1/2} - 3.95 Z T^{-1/2})$$

or $$\log_{10} \lambda = \log_{10}\left(\frac{v}{2R}\right) + 1.29 Z^{1/2} R^{1/2} - 1.72 Z T^{-1/2} \quad ...(8)$$

The changes in atomic number and nuclear radius are negligible when compared to the changes in energy. The first term is almost same for heavier nuclei. So Eq. (8) reduces to

$$\log_{10} \lambda = c + d\, T - 1/2 \quad ...(9)$$

where "c" and "d" are constants.

Eq. (9) shows that the emitters having lesser decay constants emit α-particles of greater energy (T) which is the Geiger and Nuttal law.

The Neutrino Theory of Beta Decay

In 1934, Fermi developed a theory to explain the continuous β-ray spectrum. This theory is called neutrino theory of β-decay. According to this theory, a β-particle and a neutrino are created in the nucleus and both are emitted simultaneously. The *total energy* of these two particles is a constant which is equal to the end-point energy observed in the β-ray spectrum. This maximum energy is shared by the β-particle, the neutrino and also by the recoiling nucleus. The electron will carry the maximum energy when the energy of the neutrino is zero. In all other cases electron will carry an energy less than the maximum. The sum of the energies carried by the electron and the neutrino will always be the same. This energy may be shared by the two particles in any proportion. Hence it explains the continuous β-ray spectrum.

When the nucleon shifts from the neutron quantum state to the proton quantum state, electron and antineutrino are emitted. This process is represented by

$$n \rightarrow p + e^- + \overline{\nu}\ .$$

In ordinary beta decay it is an antineutrino that is emitted.

Positron emission corresponds to the conversion of a nuclear proton into a neutron, a positron, and a neutrino.

$$p \rightarrow n + e^{+} + \nu.$$

Positron emission leads to a daughter nucleus of lower atomic number *Z* while leaving the mass number *A* unchanged. Thus negative and positive beta decays may be represented as

$$Z^{X^A} \rightarrow Z+1^{X^A} + e^{-} + \overline{\nu}$$

$$Z^{X^A} \rightarrow Z-1^{X^A} + e^{+} + \nu$$

The electron, neutrino and product nucleus share among them the energy, angular momentum and linear momentum available from the nuclear transitions. Thus the neutrino theory of β-decay successfully explains the continuous energy spectrum of β-rays.

Example : *Which of the following isobars would you expect to be* β^- *active and how would it decay ? Why ?* $_{28}Ni^{64} = 63.9280u$, $_{29}Cu^{64} = 63.928u$.

SOL. We know that for a β^- decay to be possible

$$_ZM^A > {}_{Z+1}M^A + {}_{-1}e^0 \text{ (Nuclear masses).}$$

By adding Z electron masses to both sides, the nuclear masses are changed into isotopic masses. The same condition now appears as

$$_ZM^A > {}_{Z+1}M^A \text{ (Atomic masses).}$$

The given isobars are β^- emitters, *i.e.*,

$$_{28}Ni^{64} \rightarrow {}_{29}Cu^{64} + \beta^- \text{ and } {}_{29}Cu^{64} \rightarrow {}_{30}Zn^{64} + \beta^-$$

if $_ZM^A > {}_{Z+1}M^A$. Now mass of Cu^{64} is greater than that of Ni^{64}. Hence Ni^{64} is not a β^- emitter.

Mass of Cu^{64} is greater than that of $_{30}Zn^{64}$ (63.92915*u*). Hence Cu^{64} is a β^- emitter.

Properties of Neutrinos

The properties of neutrinos can be summarized as follows:

i. Neutrinos belong to the family of leptons, and this family interacts through the weak force.
ii. There are three types or leptonic flavours of Neutrino: electron neutrino, muon neutrino and tau neutrino.
iii. A neutrino likewise has an antimatter component that is known as an antineutrino.
iv. The electromagnetic forces do not affect Neutrinos and hence, do not induce the ionization of matter.
v. Neutrinos interact with matter only through weak interactive forces.
vi. A neutrino can pass through a massive number of atoms without provoking any reaction, and hence these are the most penetrating subatomic particles.
vii. The Neutrinos can also change one nucleus into another, and this process is used in a radiochemical neutrino detector.

Gamma rays:

γ-rays from radionuclides vary in wavelength from about 0.004 to 0.4 Å. γ-rays are of the same nature as X-rays and they accompany the emission of α and β-particles in many cases. The energy of the γ-rays is very large of the order of a few MeV. The γ-rays must be of nuclear origin. γ-rays may be regarded as very short X-rays and the methods of X-ray spectroscopy can be applied, with suitable

modifications, for the determination of their wavelengths. The usual Bragg method may be adopted when the wavelength of the γ -rays exceeds 0.1 Å.

Origin of γ -rays

The large values of energy associated with γ -rays show that they must be of nuclear origin. γ -ray spectrum consists of sharp lines and this indicates the existence of a number of energy levels in the nucleus. Stable nuclides are usually in the state of least energy or ground state, but they can be excited by particle or photon bombardment. Hence nuclei can exist in states of definite energies, just as atoms can. An excited nucleus is denoted by an asterisk (*) after its usual symbol. Thus $_{38}Sr^{87}$* refers to $_{38}Sr^{87}$ in an excited state. One way an excited nucleus can return to the ground state is by the emission of γ -rays. γ -ray decay is represented schematically by

$$(\,_{Z}X^{A})^{*} \rightarrow {}_{Z}X^{A} + \gamma$$

The star (*) indicates an excited nucleus, and both the daughter and the parent have the same structure of nuclear particles. If E* is the energy associated with the excited state and E is the energy of the ground state, then the γ -rays have an energy

$$h\nu = E^{*} - E$$

where ν is the frequency of the emitted γ -ray.

A simple example of the relationship between energy levels and decay schemes is shown in Fig. 2.8 which pictures the β -decay of $_{12}Mg^{27}$ to $_{13}Al^{27}$. The half-life of the decay is 9.5 minutes, and it may take place to either of the two excited states of $_{13}Al^{27}$. The resulting $_{13}Al^{27}$* nucleus then undergoes one or two gamma decays to reach the ground state. Most excited nuclei have very short half-lives against γ -decay, but a few remain excited for as long as several hours. A long lived excited nucleus is called an isomer of the same nucleus in its ground state. The excited nucleus $_{38}Sr^{87}$* has a half-life of 2.8 hours and is an isomer of $_{38}Sr^{87}$.

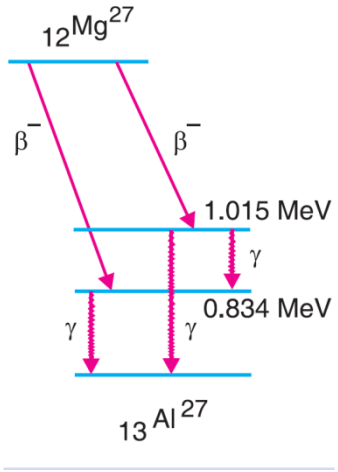


Fig 2.8

Internal Conversion

When a nucleus passes from a higher excited state to the ground state, the difference in energy of the two states is emitted as a γ -ray. As an alternative to γ -decay, an excited nucleus, in some cases, may

return to its ground state by giving up its excitation energy to one of the orbital electrons around it. The emitted electron has a K.E. equal to the lost nuclear excitation energy minus the binding energy of the electron in the atom. i.e.,

K.E. of the ejected electron = $E_e - W$.

where Ee = the available excitation energy and

W = binding energy of the ejected electron in its shell of origin.

This process is called internal conversion. The emitted electron is called a conversion electron. Thus internal conversion and emission of a γ-ray from the nucleus are two alternate ways of accomplishing the same nuclear transition. The internal conversion is not a two step process in which a γ-ray photon is first emitted and then it knocks out an orbital electron. It is in better accord with experiment to regard internal conversion as representing a direct transfer of excitation energy from a nucleus to an orbital electron.

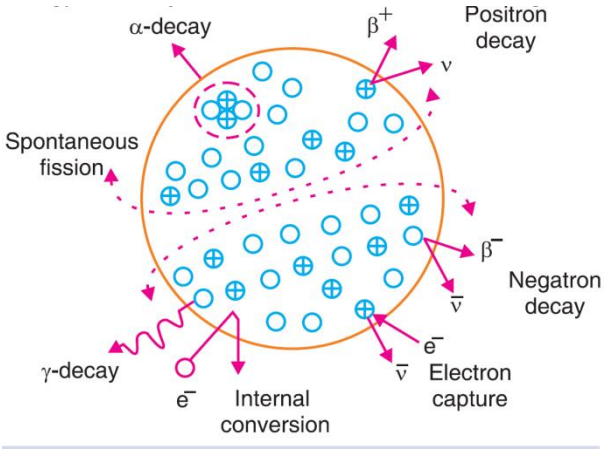


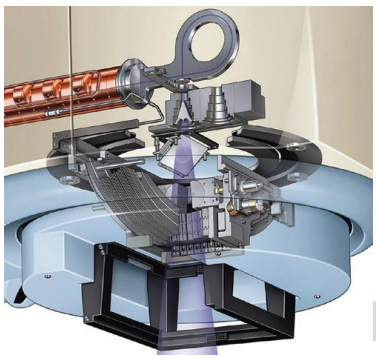
Fig 2.9

Hence, internal conversion is a single step process in which the excited nucleus interacts ejected electron (β-particle) has discrete values. Therefore, the corresponding β-particle energy spectrum is a line spectrum having discrete energies. Fig. 2.9 illustrates the various kinds of disintegration processes that radioactive nuclei may undergo. The nucleus is represented as an assembly of protons and neutrons. A proton is indicated by a cross, and a neutron by an open circle.

UNIT III – PARTICLE ACCELERATOR & DETECTORS

The Linear Accelerator:

Direct acceleration of particles by potentials above 10 million volts is a difficult problem due to insulation difficulties. For such high energies, acceleration of the particles is achieved in small successive steps. In such machines, the P.D. between different parts of the machine and between the machine and earth, is maintained low, compared with the P.D. corresponding to the ultimate energy acquired by the particles. One machine employing this method is the linear accelerator. In this machine, high energy produced without employing high P.D.'s, by using the principle of *synchronous acceleration.*



Linear accelerator

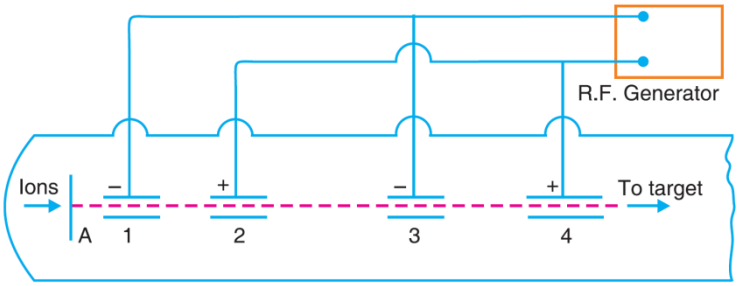


Fig 3.1

Positive ions enter along the axis of the accelerator from an ion source through an aperture *A*. Suppose a positive ion leaves *A* and is accelerated during the half cycle, when the drift tube 1 is negative with respect to *A*. Let *e* be the charge and *m* the mass of the ion and *V* potential of drift tube 1 with respect to *A*. Then velocity v_1 of the ion on reaching the drift tube is given by

$$\frac{1}{2}mv_1^2 = Ve \text{ or } v_1 = \sqrt{\frac{2Ve}{m}}.$$

The length of the tube 1 is so adjusted that as the positive ions come out of it, the tube has a positive potential and the next tube (tube No. 2) has a negative potential, *i.e.*, the potentials change sign. The positive ion is again accelerated in the space between the tubes 1 and 2. On reaching the tube 2, the velocity v_2 of the positive ion is given by

$$\frac{1}{2}mv_2^2 = 2Ve \text{ or } v_2 = \sqrt{2}\sqrt{\frac{2Ve}{m}} = \sqrt{2}\,v_1.$$

This shows that v_2 is $\sqrt{2}$ times $v1$. In order that this ion, on coming out of tube 2, may find tube 3 just negative and the tube 2 positive, it must take the same time to travel through the tube 2. Since $v_2 = \sqrt{2}v_1$, the length of tube 2 must be $\sqrt{2}$ times the length of tube 1. For successive accelerations in successive gaps the tubes 1, 2, 3, etc., must have lengths proportional to 1, $\sqrt{2}$, $\sqrt{3}$ etc. *i.e.*, $l1 : l2 : l3$: etc. = $1 : \sqrt{2}, \sqrt{3}$: etc.

Energy of the ion. If n = the number of gaps that the ion travels in the accelerator and v_n = the final velocity acquired by the ion, then

$$\left.\begin{array}{l}\text{Velocity of the ion, as it}\\ \text{emerges out of the } n^{th} \text{ tube}\end{array}\right\} = \sqrt{n}\sqrt{\frac{2Ve}{m}}.$$

$$\therefore \quad \text{K.E. acquired by the ion} = \frac{1}{2}mv_n^2 = nVe.$$

Thus the final energy of the ions depends upon (*i*) the total number of gaps and (*ii*) the energy gained in each gap. The limitations of this accelerator are : (*i*) The length of the accelerator becomes inconveniently large and it is difficult to maintain vacuum in a large chamber. (*ii*) The ion current available is in the form of short interval impulses because the ions are injected at an appropriate moment.

Example. *In a linear accelerator, proton accelerated thrice by a potential of* 40 *kV leaves a tube and enters an accelerating space of length* 30 *cm before entering the next tube. Calculate the frequency of the r. f. voltage and the length of the tube entered by the proton*

Sol. *Let v1 and v2 be the velocities of the proton on entering and leaving the accelerating space. Let e and m be the mass and charge of the proton respectively. Then*

$$\frac{1}{2}mv_1^2 = 3 \times e \times 40000$$

$$\therefore \quad v_1 = [(2 \times 3 \times 40000 \times (e/m)]^{1/2}$$
$$= [2 \times 3 \times 40000 \times (9.578 \times 10^7)]^{1/2}$$
$$= 4.794 \times 10^6\ ms^{-1}.$$

$$\text{Similarly,} \quad v_2 = \left[2\times4\times40000\times(9.578\times10^7)\right]^{1/2}$$
$$= 5.536 \times 10^6\ ms^{-1}.$$

Mean velocity while travelling the 0.3 *m* distance

$$= 5.165 \times 10^6\ ms^{-1}.$$

The time taken to travel 0.3 *m* is half the period (*T*/2) of the *r.f.* voltage.

$$\therefore \quad \frac{T}{2} = \frac{0.3}{5.165\times10^6} \text{ or } T = \frac{2\times0.3}{5.165\times10^6}$$

$$\therefore \quad \left.\begin{array}{l}\text{frequency of the}\\ r.f.\text{voltage}\end{array}\right\} = \frac{5.165\times10^6}{2\times0.3}$$

$$= 8.608 \times 10^6\ Hz = 8.608\ MHz.$$

The protons travel through the next tube for half a period with a velocity of $5.536 \times 10^6\ ms^{-1}$.

$\therefore$ length of the tube entered by the protons

$$L = 5.536\times10^6\times\frac{1}{2\times8.608\times10^6} = 0.3216\ m$$

The Cyclotron:

Construction. The cyclotron (Fig. 3.2) consists of two hollow semi-circular metal boxes, *D*1, *D*2 called ‘’dees’’. A source of ions is located near the mid-point of the gap between the ‘’dees’’ . The “dees’’ are insulated from each other and are enclosed in another vacuum chamber. The “dees’’ are

connected to a powerful radio-frequency oscillator. The whole apparatus is placed between the pole-pieces of a strong electromagnet. The magnetic field is perpendicular to the plane of the "dees''.

Theory. Suppose a positive ion leaves the ion source at the centre of the chamber at the instant when the "dees'' *D*1 and *D*2 are at the maximum negative and positive A.C. potentials respectively. The positive ion will be accelerated towards the negative dee *D*1 before entering it. The ions enter the space inside the dee with a velocity *v* given by , $Ve = 1/2\ mv^2$ where *V* is the applied voltage and *e* and *m* are the charge and mass of the ion respectively. When the ion is inside the "dee" it is not accelerated since this space is field free. Inside the dee, under the action of the applied magnetic field, the ions travel in a circular path of radius *r* given by

$$Bev = mv^{2/r}$$

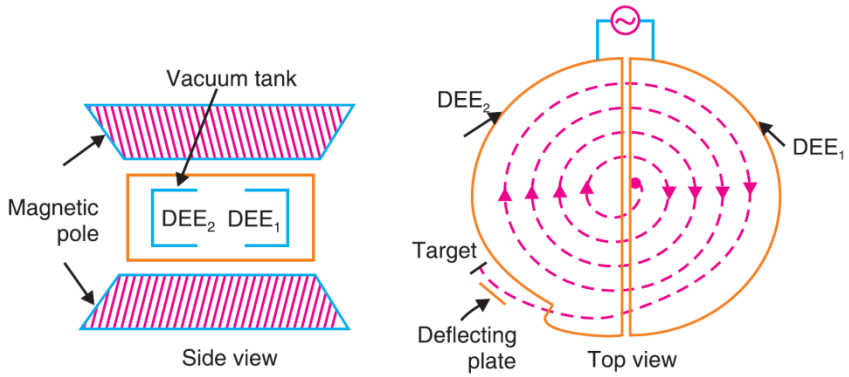


Fig 3.2

where *B* = the flux density of the magnetic field.

$$\text{or } r = mv/Be \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ldots(2)$$

$$\text{The angular velocity of the ion in its circular path} = \omega = \frac{v}{r} = \frac{Be}{m} \qquad \ldots(3)$$

$$\text{The time taken by the ion to travel the semicircular path} = t = \frac{\pi}{\omega} = \frac{\pi m}{Be} \qquad \ldots(4)$$

Suppose the strength of the field (*B*) or the frequency of the oscillator (*f*) are so adjusted that by the time the ion has described a semi-circular path and just enters the space between *D*1 and *D*2, *D*2 has become negative with respect to *D*1. The ion is then accelerated towards *D*2 and enters the space inside it with a greater velocity. Since the ion is now moving with greater velocity, it will describe a semicircle of greater radius in the second "dee''. But from the equation $t = \pi m/Be$ it is clear that the time taken by the ion to describe a semicircle is independent of both the radius of the path (*r*) and the velocity of the ion (*v*). Hence the ion describes all semicircles, whatever be their radii, in exactly the same time. This process continues until the ion reaches the periphery of the dees. The ion thus spirals round in circles of increasing radius and acquires high energy. The ion will finally come out of the dees in the direction indicated, through the window.

Energy of an ion. Let r_{max} be the radius of the outermost orbit described by the ion and *vmax* the maximum velocity gained by the ion in its final orbit. Then the equation for the motion of the ion in a magnetic field is

$$Bev_{max} = \frac{mv_{max}^2}{r_{max}}$$

or $$v_{max} = B\frac{e}{m}r_{max} \quad \text{..} \quad)$$

$\therefore$ The energy of the ion

$\therefore$ $$E = \frac{1}{2}mv_{max}^2 = \frac{B^2 r_{max}^2}{2}\left[\frac{e^2}{m}\right] \quad \text{..} \quad)$$

The condition for acceleration of the ion in the inter-dee gap is that

The time taken by the ion to travel the semicircular path } = Half the time period of oscillation of the applied high frequency voltage

.

$$i.e., \quad \frac{\pi m}{Be} = \frac{T}{2} \text{ or } T = \frac{2\pi m}{Be}$$

$\therefore$ Frequency of the oscillator

$$f = \frac{Be}{2\pi m}$$

Hence the energy of the ion is given by

$$E = 2\pi^2 r_{max}^2 f^2 m$$

The particles are ejected out of the cyclotron not continuously but as pulsed streams.

Limitations of the Cyclotron. The energies to which particles can be accelerated in a cyclotron are limited by the relativistic increase of mass with velocity. The mass of a particle, when moving with a velocity v is given by m= $\frac{m0}{\sqrt{1-v2/c2}}$ where m_o is the rest mass and c the velocity of light.

According to equation (4),

$$\left.\begin{array}{l}\text{The time taken by the ion to}\\ \text{travel the semicircular path}\end{array}\right\} = t = \frac{\pi m}{Be} = \frac{T}{2}$$

$\therefore$ Frequency of the ion} $= n = \frac{1}{T} = \frac{Be}{2\pi m}$ or $n = \frac{Be\sqrt{1-v^2/c^2}}{2\pi m_0}$.

Therefore, the frequency of rotation of the ion decreases with increase in velocity. The ions take longer time to describe their semicircular paths than the fixed period of the oscillating electric field. Thus, the ions lag behind the applied potential and finally they are not accelerated further. Due to this reason, the energy of the ions produced by the cyclotron is limited. This limitation can be overcome in the following two ways.

Now, the frequency of the ion = $Be\frac{\sqrt{1-v^2/c^2}}{2\pi m_0}$.

(*i*) Field variation. The frequency of the ion can be kept constant by increasing the magnetic field (B) at such a rate that the product $B\ 1-v2\ /\ c2$ remains constant. For this purpose, the value of the magnetic field B should increase, as velocity of the ion increases, so that the product $B\ 1-v2\ /\ c2$ remains unchanged. This type of machine in which the frequency of electric field is kept constant and magnetic field is varied is called *synchrotron*

(*ii*) Frequency modulation. In another form of apparatus, the frequency of the applied A.C. is varied so that it is always equal to the frequency of rotation of the ion. This type of machine in which magnetic field is kept constant and the frequency of the applied electric field is varied is called a *frequency modulated cyclotron* or *synchro-cyclotron*

Example 1. *A cyclotron in which the flux density is 1.4 weber/m2 is employed to accelerate protons. How rapidly should the electric field between the dees be reversed? Mass of the proton* $= 1.67 \times 10^{-27}$ *kg and charge* $= 1.6 \times 10^{-19}$ *C.*

SOL. Here, $B = 1.4$ weber/m^2; $m = 1.67 \times 10^{-27}$ kg; $e = 1.6 \times 10^{-19}$ C.

$$\therefore \qquad t = \frac{\pi m}{Be} = \frac{\pi(1.67\times10^{-27})}{1.4\times(1.6\times10^{-19})} = 2.342\times10^{-8}\ s.$$

Example 2. *Deuterons in a cyclotron describe a circle of radius 0.32 m just before emerging from the dees. The frequency of the applied e.m.f. is 10 MHz. Find the flux density of the magnetic field and the velocity of deuterons emerging out of the cyclotron. Mass of deuterium* $= 3.32 \times 10^{-27}$ *kg;* $e = 1.6 \times 10^{-19}$ *C.*

SOL. We have, $f = \dfrac{Be}{2\pi m} \quad \therefore\ B = \dfrac{2\pi mf}{e}$

Here, $m = 3.32 \times 10^{-27}$ kg; $f = 10\ MHz = 10^7\ Hz$; $e = 1.6 \times 10^{-19}\ C$

$$\therefore \qquad B = \frac{2\pi(3.32\times10^{-27})10^7}{1.6\times10^{-19}} = 1.303\ \text{weber/m}^2$$

We have $\dfrac{mv^2}{r_{max}} = Bev$ or $v = \dfrac{Be\,r_{max}}{m}$

Here, $B = 1.303$ weber/m^2 ; $e = 1.6 \times 10^{-19}$ C; $r_{max} = 0.32$ m and $m = 3.32 \times 10^{-27}$ kg.

$$v = \frac{Be\,r_{max}}{m} = \frac{1.303(1.6\times10^{-19})0.32}{3.32\times10^{-27}}$$

$$= 2.009 \times 10^7\ \text{ms}^{-1}.$$

The Betatron : Betatron is a device to accelerate electrons (beta particles) to very high energies. It was constructed in 1941 by D.W. Kerst. The action of this device depends on the principle of a transformer.

Construction: It consists of a doughnut—shaped vacuum chamber placed between the pole-pieces of an electromagnet. The electromagnet is energised by an alternating current. The magnet produces a strong magnetic field in the doughnut. The electrons are produced by the electron gun (FG) and are allowed to move in a circular orbit of constant radius in the vacuum chamber (Fig. 3.4). The magnetic field varies very slowly compared with the frequency of revolution of the electrons in the equilibrium orbit.

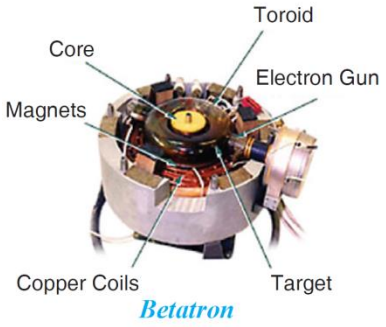
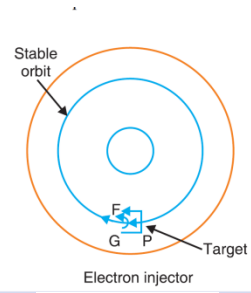


Fig 3.3



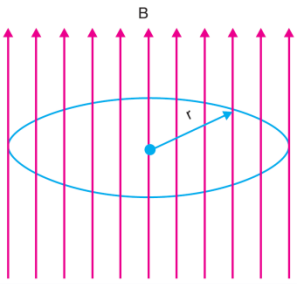


Fig 3.4

The varying magnetic field, acting parallel to the axis of the vacuum tube, produces two effects on the elect s viz., (*i*) The changing flux due to the electromagnet produces the induced e.m.f. which is responsible fo e acceleration of the electrons. (*ii*) The field of the magnet serves at the same time to bend the electrons in a circ r path in the chamber and confine them to the region of the changing flux.

Theory. Consider the electron moving in an orbit of radius *r* (Fig. 30.9). Let ϕ be the flux linked with the o t. The flux increases at the rate $d\phi/dt$ and the induced e.m.f. in the orbit is given by

$$E = -\frac{d\phi}{dt} \qquad ...(1)$$

The work done on an electron of charge *e* in one revolution = E*e* = $-e\frac{d\phi}{dt}$ ...(2)

Let *F* be the tangential electric force acting on the orbiting electron.
For one revolution, the path length is $2\pi r$.
Then the work done on the electron in one revolution = $F \times 2\pi r$

$$\therefore \quad F \times 2\pi r = -e\frac{d\phi}{dt}$$

or
$$F = -\frac{e}{2\pi r}\frac{d\phi}{dt} \qquad ...(3)$$

When the velocity of the electron increases due to the above force, it will try to move into an orbit of larger radius. Because of the presence of the magnetic flux perpendicular to the plane of the electron orbit, the electron will experience a radial force inward given by

$$Bev = mv^2/r \qquad ...(4)$$

Here *B* is the value of the magnetic field intensity at the electron orbit of constant radius *r*, *v* = velocity of the electron and *m* = mass of the electron. From (4),
the momentum of the electron = *mv* = *Ber* ...(5)
From Newton's second law of motion,

$$F = \frac{d}{dt}(mv) = er\frac{dB}{dt} \qquad ...(6)$$

To maintain the constant radius of the orbit, the values of *F* given in equations (3) and (6) must be numerically equal.

$$\therefore \qquad \frac{e}{2\pi r}\frac{d\phi}{dt} = er\frac{dB}{dt} \text{ or } d\phi = 2\pi r^2\, dB$$

$$\text{Integrating, } \int_0^{\phi} d\phi = \int_0^{B} 2\pi r^2\, dB$$

or $$\phi = 2\pi r^2 B \qquad ...(7)$$

If the uniform magnetic field B acts over an area πr^2, the magnetic flux $\phi' = \pi r^2 B$. Therefore the flux through the orbit is twice the flux enclosed by the orbit, if the magnetic field were to be uniform over the area. Equation (7) represents the condition under which a betatron works and is called *betatron* condition. This distribution of magnetic flux is obtained by the special pole-pieces where the magnetic field is greater at the centre of the orbit than at its circumference.

Fig. 30.4 shows the variation of magnetic field with time.

Electrons are injected into the chamber when magnetic field just begins to rise. The electrons are then accelerated by the increasing magnetic flux linked with the electron orbit. During the time the magnetic field reaches its peak value, the electrons make several thousand revolutions and get accelerated. If they are allowed to revolve any more, the decreasing magnetic field would retard the electrons. Hence, the electrons are extracted at this stage by using an auxiliary magnetic field to deflect them from their normal course. The high energy electron beam can be made to strike the target, generating X-rays. Alternately the electrons can be made to emerge out of the apparatus and used for transmutation work.

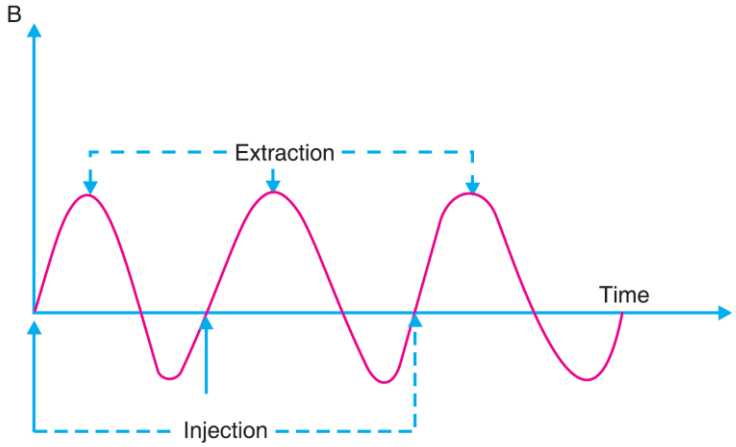


Fig 3.4

Example. In a certain betatron the maximum magnetic field at orbit was 0.4 Wb/m^2, operating at 50 Hz with a stable orbit diameter of 1.524 m. Calculate the average energy gained per revolution and the final energy of the electrons.

SOL. In the betatron, the electron velocities are nearly c.

$\therefore$ the total distance travelled in the acceleration time (*i.e.*, one quarter cycle) $= c\times\frac{T}{4} = c\times\frac{\pi}{2\omega}$

Total number of revolutions

$$= N = \frac{c\pi/2\omega}{2\pi r} = \frac{c}{4\omega r}.$$

Here, frequency $= f = 50$ Hz. $\therefore\ \omega = 2\pi f = 2\pi \times 50 = 100\ \pi$; $r = 0.762$ m, and $c = 3 \times 10^8\ ms^{-1}$.

$$\therefore \qquad N = \frac{3\times10^8}{4(100\pi)0.762} = 3.132\times10^5$$

Let E be the final energy acquired by the electrons. Since the electrons must be treated relativistically,

momentum of the electron $= mv = E/c$

But $mv^2/r = Bev$ or $mv = Ber$ or $E = Berc$.

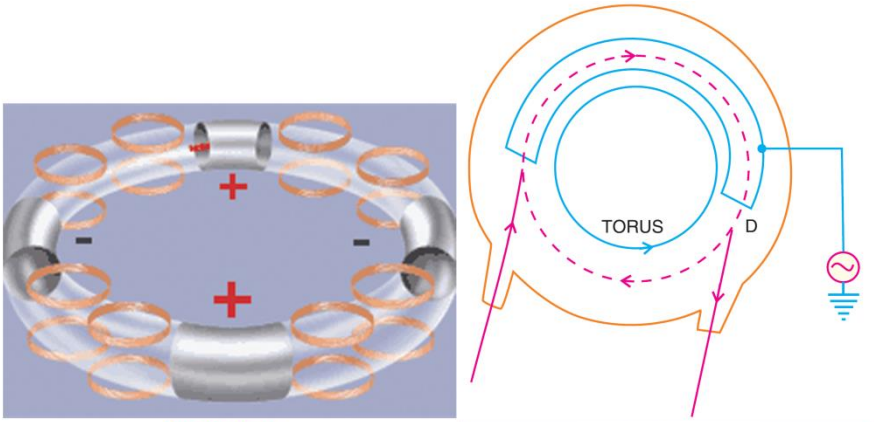
$$\therefore \qquad E = \frac{0.4(1.6\times10^{-19})(0.762)(3\times10^8)}{1.6\times10^{-13}}\ MeV$$

$$= 91.45\ MeV.$$

$$\text{Average energy gained per revolution} = \frac{91.45\times10^6}{3.132\times10^5} = 291.9\,eV.$$

Electron synchrotron:

The electron synchrotron is based on the principle of the combined working of betatron and cyclotron. Electrons are injected into an orbit of fixed radius at an initial energy of about 50 to 80 keV. The main accelerating tube, the torus, is made of glass or some plastic with a circular "dee" (D) made of a metal. An alternating potential is applied to the "dee" as shown in Fig. 3.5. A varying magnetic field is applied perpendicular to the torus. The radius of the orbit is kept constant by increasing the magnetic field as in a betatron. The increments of energy are given, as in a cyclotron, at the beginning and ending of the D. The electrons, after acceleration, are made to strike the required target. Using tungsten as target, very hard X-rays of energy about 300 MeV have been produced.



Synchrotrons. **Fig 3.5**

Radiation detectors:

Most of the nuclear reactions are accompanied by the emission of charged particles like α-particles, protons, electrons and radiations like γ-rays. In order to understand these particles and their interaction with atomic nuclei, precise information about their mass, momentum, energy, etc., are necessary. We shall describe in this chapter some of the common techniques employed for the detection of nuclear radiations and for analysing their energies. Several nuclear radiation detectors depend for their operation on the *ionization* that is produced in them by the passage of charged particles. This group of detectors includes ionization chambers, proportional counters, G-M counters, semiconductor radiation detectors, cloud chambers and spark chambers. In other detectors, excitation and sometimes molecular dissociation also play important roles. These phenomena, in combination with ionization, bring about the luminescence in scintillation detectors and the latent images in photographic emulsions.

Ionization Chamber

The principle employed here is that charged subatomic particles can ionise gases. The number of ion-pairs produced gives us information not only on the nature of the incident particles, but even on their energy. The ionisation chamber consists of a hollow metallic cylinder *C*, closed at both ends, with a window *W* at an end for the entry of the ionising particles or radiation [Fig.3.6]. A metal rod *R*, well insulated from the cylinder, is mounted coaxially within the cylinder. *R* is connected to a quadrant electrometer *E*. A p.d. of several hundred volts is maintained between *C* and *R*. An earthed guard ring *G* prevents leakage of charge from the cylinder to the rod. The chamber contains some gas like sulphur dioxide or methyl bromide.

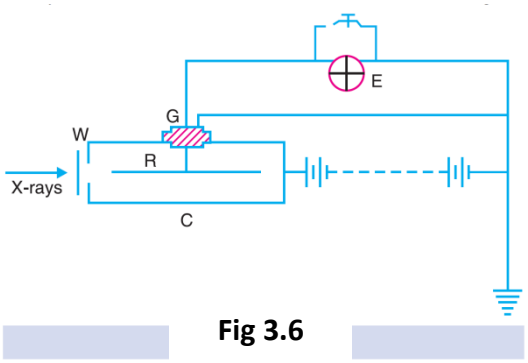


Fig 3.6

When a charged particle enters the chamber, it produces a large number of ion pairs in the enclosed gas, along its path. Positive ions move towards *R* and negative ions towards *C*. The quadrant electrometer *E* measures the rate of deposition of positive charges on *R*. The ionisation currents produced are quite small $\approx 10^{-12} - 10^{-15}$ amperes. Special electrometers and D.C. amplifying devices have to be employed to measure such small currents. If individual particles are to be counted, then the pulses of current produced are fed to a pulse amplifier, which is joined to the ionisation chamber by a coupling capacitor [Fig. 3.7]. Ionisation chambers have been used to study α-particles, β-particles, protons, electrons and nuclei of lighter elements. Ionisation chambers were extensively used in the early studies of cosmic ray phenomena. Ionisation chambers can also be used for measurements on X-rays and γ-rays. For neutron detection, the chamber is filled with boron trifluoride vapour (where the boron is enriched with B^{10}) or the chamber walls are lined inside with a boron compound in the form of a paste.

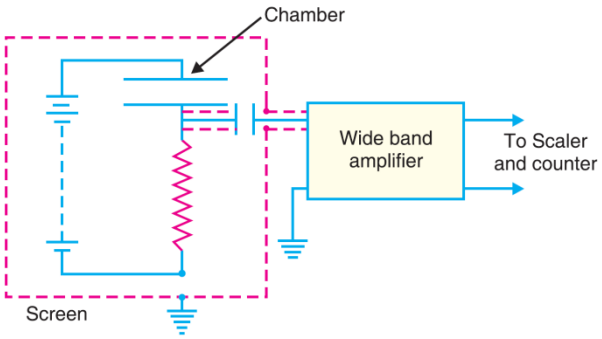


Fig 3.7

An ionisation chamber is much less sensitive to β-particles (in comparison to α-particles) because β-particles produce fewer pairs of ions in their passage through the chamber.

For detecting γ-rays, an ionisation chamber of thick wall made of high atomic-number material (Pt, Bi) is employed. The γ-rays impinging on the *walls* of the chamber eject high-speed electrons which produce ionisation in the gas.

Example 1. α-particles of energy 5 MeV pass through an ionisation chamber at the rate of 10 per second. Assuming all the energy is used in producing ion pairs, calculate the current produced. (35 eV is required for producing an ion pair and e = 1.6×10^{-19} C).

SOL. Energy of α-particles = $5 \times 10^6\ eV$.

Energy required for producing one ion pair $= 35\ eV$

No. of ion pairs produced by one α-particle

$$= \frac{5\times10^6}{35} = 1.429\times10^5$$

Since 10 particles enter the chamber in one second,

No. of ion pairs produced per second

$$= 1.429 \times 10^5 \times 10 = 1.429 \times 10^6.$$

$$\text{Charge on each ion} = 1.6 \times 10^{-19}\ C.$$

$$\therefore \quad \text{Current} = (1.429 \times 10^6) \times (1.6 \times 10^{-19})\ C/s$$

$$= 2.287 \times 10^{-13}\ A.$$

Example 2. An ionization chamber is connected to an electrometer of capacitance 0.5 pF and voltage sensitivity of 4 divisions per volt. A beam of α-particles causes a deflection of 0.8 divisions. Calculate the number of ion pairs required and the energy of the α-particles. Given that 1 ion pair requires energy of 35 eV and e = 1.6 × 10– 19 coulomb.

Sol. Voltage sensitivity of electrometer = 4 divisions/volt.

$$\therefore \quad \left.\begin{matrix}\text{Voltage required to produce}\\ \text{a deflection of 0.8 divisions}\end{matrix}\right\} = \frac{0.8}{4} \text{ volt} = 0.2 \text{ volt}$$

$$Q = CV = (0.5 \times 10^{-12}) \times 0.2 \text{ [since } C = 0.5\, pF = 0.5 \times 10^{-12}\, F\text{]}$$
$$= 10^{-13}\, C.$$

$$\therefore \quad \text{No. of ion pairs required} = \frac{10^{-13}}{1.6\times10^{-19}} = 6.25\times10^{5}$$

1 ion pair requires 35 *eV*.

$$\therefore \quad \text{Total energy required} = 35 \times (6.25 \times 10^{5}) \text{ eV}$$
$$= 21.88 \text{ MeV}.$$

Proportional Counter

The proportional counter consists of a cylindrical gas filled tube with a very thin central wire which serves as the anode (Fig. 3.8). The outer cylinder serves as a cathode. In the case of the simple ionization chamber, the pulse height generated by an event is proportional to the intensity of the beam. But because of the comparatively low applied voltages, the current produced is always very small. If the voltage applied to an ionization chamber is increased past a certain value, the electrons acquire enough energy while moving toward the anode to create further ion pairs along the way. The resulting avalanche of secondary electrons that reaches the anode may represent a multiplication factor of as much as 1,000 with a correspondingly larger output pulse. In a certain range of applied voltages (See Fig. 3.9), the pulse size is proportional to the original number of ion pairs, and the device is called a proportional counter. Since the central wire is very thin and the p.d. fairly large, the electric field $E = dV/dr$ at a distance r from the centre is very high. If b is the radius of the cylinder and a the radius of the wire, the radial field E at a distance r from the centre is given by

$$E = \frac{V}{r \log_e (b/a)}$$

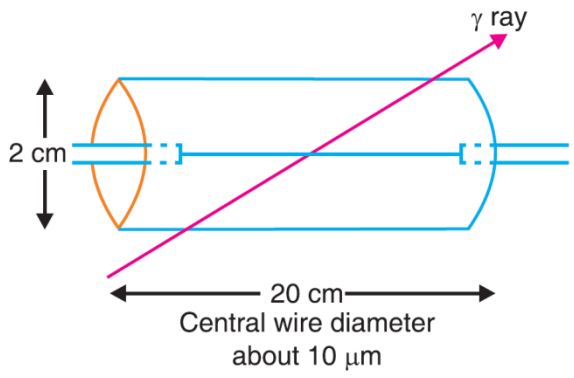


Fig 3.8

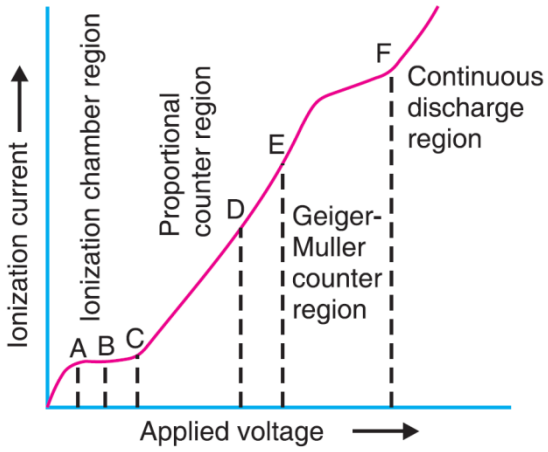


Fig 3.9

where V is the positive voltage of the central wire relative to the outer cylinder. Thus, in a proportional counter, the field strength near the wire is very great. Hence electrons travelling towards the wire are rapidly accelerated when near it, and produce additional electrons in that region due to the phenomena of ionization by

collision. This process is called the *gas multiplication.* The complete voltage pulse characteristics of this type of tube are shown in Fig. 3.9. The main regions used for measurements are : (1) The ionization chamber regio *AB* (2) the proportional counter region *CD* (3) the Geiger-Muller region *EF*. After the point *F*, the tube becomes a simple discharge tube in which the current is produced even after the ionization event has ceased. Like the ionization chamber, the proportional counter gives single pulses of height proportional to the ionizing power of the radiation.

Example. It is required to operate a proportional counter with a maximum radial field of 10^7 Vm^{-1}. What is the applied voltage required if the radii of the wire and tube are 0.002 cm and 1 cm respectively?

Sol. Radial field $= E = \dfrac{V}{r \log_e (b/a)}$

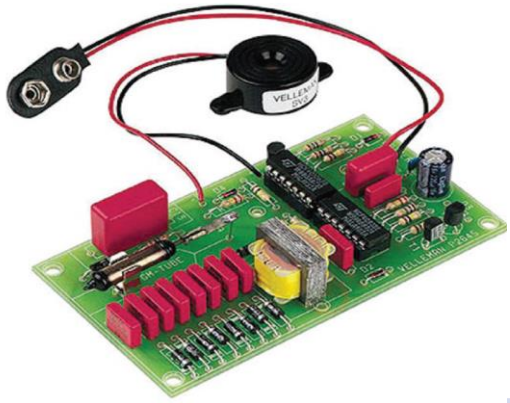
Radial field at the wire surface $E = \dfrac{V}{a \log_e (b/a)}$

$$\therefore \qquad 10^7 = \frac{V}{\left(2\times10^{-5}\right) 2.302 \log_{10}\left(10^{-2}/2\times10^{-5}\right)}$$

or $\qquad V = 1242$ volts.

Geiger-Muller Counter :

It consists of a metal chamber C containing air or some other gas at a pressure of about 10 cm of Hg. A fin tungsten wire (W) is stretched along the axis of the tube and is insulated from it by ebonite plugs EE (Fig 3.10). The wire is connected to the positive terminal of a high tension battery (about 1000 to 3000 volts through a high resistance R (about 100 megohms) and the negative terminal is connected to the chamber C The D.C. Voltage is kept slightly less than that which will cause a discharge between the electrodes.



Geiger-Muller Counter.

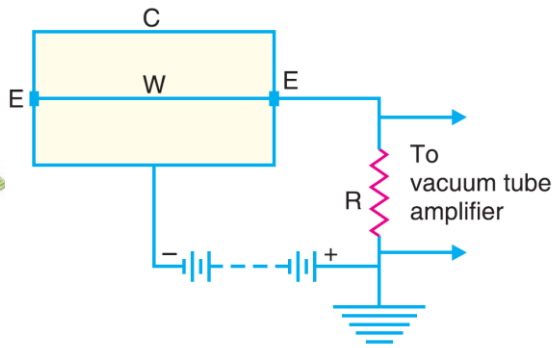


Fig 3.10

When an ionizing particle (say an α particle) enters the counter, ionisation takes place and a few ions ar produced. If the applied P.D. is strong enough, these ions are multiplied by further collisions. An avalanche o electrons moves towards the central wire and this is equivalent to a small current impulse which flows throug the resistance *R*. The critical potential is lowered momentarily, causing a sudden discharge through th resistance *R*. The p.d. thus developed across *R* is amplified by vacuum tube circuits and is made to operate mechanical counter. In this way single particles can be registered. The sudden pulse of discharge sweeps awa the ions from the chamber and the counter is ready to register the arrival of the next particle.

The voltage characteristics of a Geiger-Muller counter are shown in Fig. 3.11. This is a plot of the countin rate against the counter potential with a radioactive source placed near the counter. It is seen that there is

threshold below which the tube does not work. This can be several hundred volts. As the applied potential i increased, the counting begins and rises rapidly to a flat portion of the curve called the *plateau.* This is th region of the counter operation where the counting rate is, more or less, independent of small changes in p.c across the tube. Beyond the plateau the applied electric field is so high that a continuous discharge takes plac in the tube as shown in Fig. 3.11 and the count rate increases very rapidly. It does not require any ionizin event for this to happen so that the tube must not be used in this region.

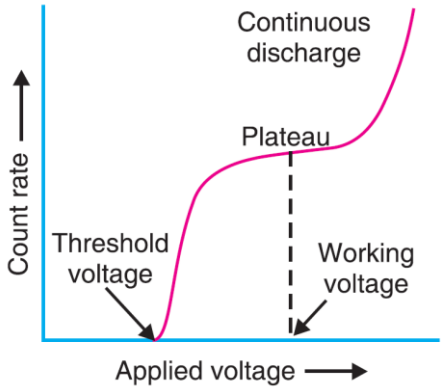


Fig 3.11

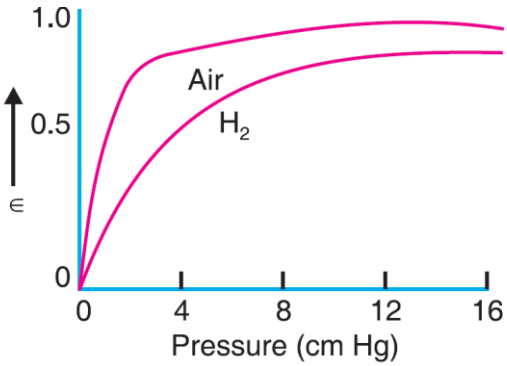


Fig 3.12

The *efficiency of the counter* is defined as the ratio of the observed counts/sec. to the number of ionizin particles entering the counter per second. Counting efficiency is defined as the ability of its counting, if at leas one ion pair is produced in it. Counting efficiency = $\in = 1 - e^{slp}$ where s = specific ionization at on atmosphere; p = pressure in atmospheres and l = path length of the ionization particle in the counter. Th efficiency ε of a *GM* counter, as a function of pressure for air and hydrogen, is illustrated in Fig. 3.12. Th counter set-up is portable (with the transistorised electronics) and serves best for mineral prospecting, apa from its several other applications in cosmic ray work. A virtue of the Geiger counter is that the pulse height i constant over a range of applied voltages, as in Fig. 3.12. So the power supply does not have to be precisel regulated as it does for a proportional counter. Also, the pulses are several volts in height, which make amplifiers unnecessary.

Disadvantages of the Geiger counter are: (*i*) it is insensitive for a period of 200 to 400 μ s following eac pulse, which prevents its use at very high counting rates. (*ii*) it cannot provide information about the particle c photon causing a pulse.

Example. *A self-quenched G-M counter operates at* 1000 *volts and has a wire diameter of* 0.2 *mm. The radiu of the cathode is* 2 *cm and the tube has a guaranteed lifetime of* 10^9 *counts. What is the maximum radial fiel and how long will the counter last if it is used on an average for* 30 *hours per week at* 3000 *counts per minute Consider 50 weeks to a year.*

SOL. The radial field at the central wire is

$$E_{\max} = \frac{V}{r\log_e(b/a)} = \frac{1000}{0.0001\times 2.3026\log_{10}\left(\frac{2\times 10^{-2}}{10^{-4}}\right)}$$

$$= 1.89 \times 10^6 \text{ volts/metre.}$$

If the lifetime of the tube is N years, the total number of counts recorded will be

$$N \times 50 \times 30 \times 60 \times 3000 = 2.7 \times 10^8 N$$

$$\therefore \quad 2.7 \times 10^8 \times N = 10^9$$

or $$N = 3.7 \text{ years.}$$

The Wilson Cloud Chamber

Principle. If there is a sudden expansion of saturated vapour in a chamber, supercooling of the vapour occurs Tiny droplets will be formed by condensation over the dust particles present in the chamber. If, therefore, w have completely dust-free and saturated air, and if it is suddenly allowed to expand and thereby coo condensation will not take place. But if ions are available in the chamber during the expansion, they serve a nuclei for condensation. Hence, if an ionising particle passes through the chamber during an expansion, ion are produced along its path and droplets condense on these ions. Hence the "track" of the particle become visible.

Description. The apparatus consists of a large cylindrical chamber *A*, with walls and ceiling made of glas (Fig. 3.13). It contains dust-free air saturated with water vapour. *P* is a piston working inside the chambe When the piston moves down rapidly, adiabatic expansion of the air inside the chamber takes place. The pisto is connected to a large evacuated vessel *F* through a valve *V*. When the valve is opened, the air under th piston rushes into the evacuated vessel *F*, thereby causing the piston to drop suddenly. The wooden blocks *W* reduce the air space inside the piston. Water at the bottom of the apparatus ensures saturation in the chambe The expansion ratio can be adjusted by altering the height of the piston.

As soon as the gas in the expansion chamber is subjected to sudden expansion, the ionizing particles ar shot into the chamber through a side window. A large number of extremely fine droplets are formed on all th ions produced by the ionizing particles. These droplets form a track of the moving ionizing particles. At thi stage, the expansion chamber is profusely illuminated by a powerful beam of light *L*. Two cameras *CC* ar used to photograph the tracks. The process of expansion, shooting of the ionising particles into the expansio chamber, illuminating the chamber and clicking the camera must all be carried out in rapid succession in orde to get satisfactory results. The ionising agent can be easily identified from its path in the cloud chamber. α particles, being comparatively massive, go straight and their paths are *thick, straight* and *sharply defined.* β particles being lighter, are easily deflected by collision and their paths are thin and *crooked.* The clou chamber has led to the discovery of many elementary particles like positron, meson, etc.

Wilson Cloud Chamber

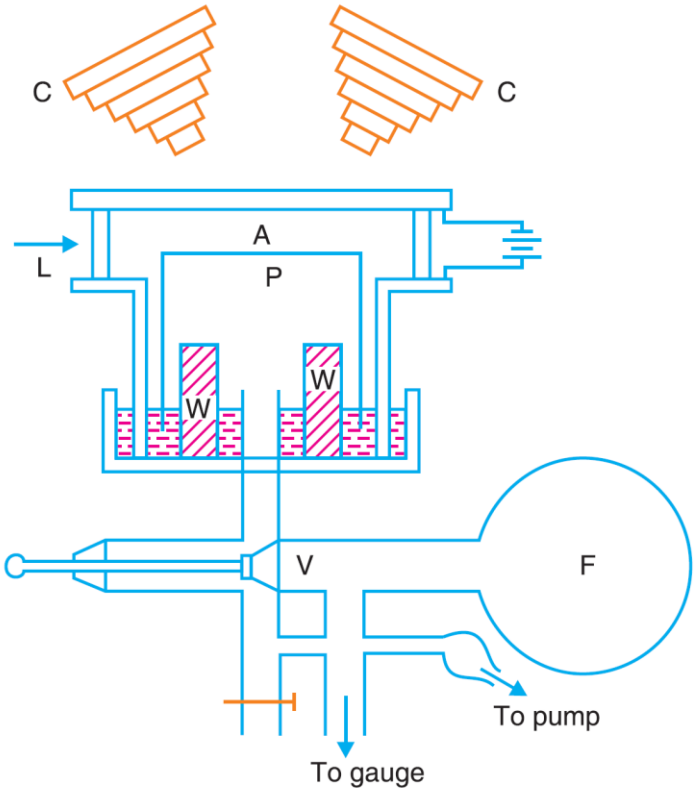


Fig 3.13

Advantages . (1) Cloud chambers can be used to study the variation of specific ionisation along the track of charged particle and the range of such particles. (2) The sign of the electric charge and the momentum *p* of th particle can be determined if the chamber is placed in a strong magnetic field. Let a particle of mass *m* an charge *q* move with a velocity *v* perpendicular to the direction of the magnetic field of flux density *B*. Th particle will be forced by the field to follow a circular path of radius *R*. The magnetic force *Bqv* is exactl balanced by the centrifugal force mv^2/R.

Thus $Bqv = Mv^2/R$ or $mv = p = BRq$.

The K.E. of the particle can be calculated, if the rest mass energy $m0c2$ of the particle is known, by th relation,

$$K.E. = E_k = \sqrt{\left[p^2c^2 + \left(m_0c^2\right)^2\right]} - m_0c^2$$

Limitations.

(*i*) One is not always sure of the sense of track photographed.

(*ii*) The range of the particle may exceed the dimensions of the chamber so that the whole track is nc photographed.

(*iii*) There remains a certain amount of uncertainty about the nature of the nuclei constituting the arms of th forked tracks.

Diffusion Cloud Chamber

The disadvantage of the cloud chamber lies in the fact that it needs a definite time to recover after a expansion. Hence it is not possible to have a continuous record of events taking place in the chamber. Thi difficulty was removed by the introduction of the diffusion cloud chamber. The outline of the apparatus i shown in Fig. 3.14. It consists of a chamber containing a heavy gas which is kept warm at the top and colda the bottom. Thermal gradient is maintained between the bottom and top of the chamber by external heating o cooling. The liquid (methyl alcohol) vaporises in the warm region, where the vapour pressure is high. Th vapour diffuses downwards continuously where the vapour pressure is low and condensation takes place. In region near the base, the super saturation factor is high and condensation takes place around the available ions The chamber remains continuously sensitive to ionizing particles until the supply of volatile liquid i exhausted. The system is illuminated by a strong source of light and the track of the particle is photographe by camera.

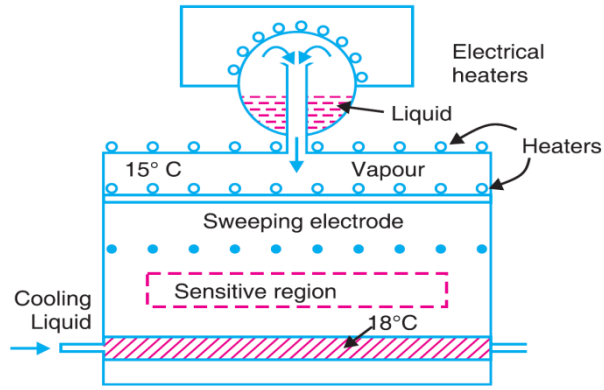


Fig 3.14

The Scintillation Counters

One of the earliest methods of radiation detection was the spinthariscope (Fig. 3.15). It consists of a smal wire, the tip of which is dipped in Radium bromide (*R*) or any other radioactive salt. It is placed in front of zinc sulphide screen *S* and viewed through a microscope. When an α or β-particle falls on the zinc sulphid screen, they produce light flashes which can be seen by a microscope (*M*) in a dark room. The visibl luminescence excited in zinc sulphide by α-particles was used by Rutherford for counting the particles. Th process of counting these scintillations through a low power microscope is a tedious one and the limitations o observation with the eye restrict the counting rate to about 100 per minute. This process, whereby the energ of the particle is converted to light, is the basis of scintillation counter.

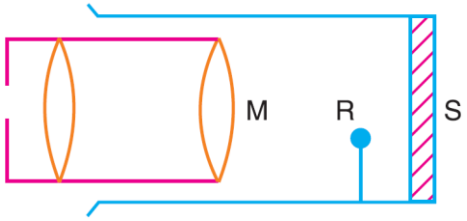


Fig 3.15

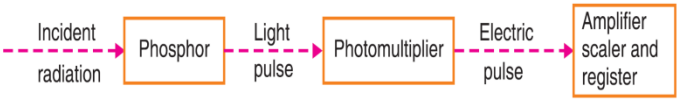


Fig 3.16

The main parts of a scintillation counter are shown in Fig. 3.16. The atoms of the phosphor are excited c ionised by the energy loss of an impinging α, β or γ ray. When the atoms return to their ground state photons are emitted, in the blue and ultraviolet regions of the optical spectrum. The phosphor is opticall coupled to the envelope of a photomultiplier tube. The photons strike the photocathode, causing the ejection c photo-electrons (Fig. 3.17). As these photo-electrons leave the photocathode, they are directed by a focusin electrode to the first multiplier electrode or *dynode*. This electrode has the property of emitting three, four c five electrons for every single

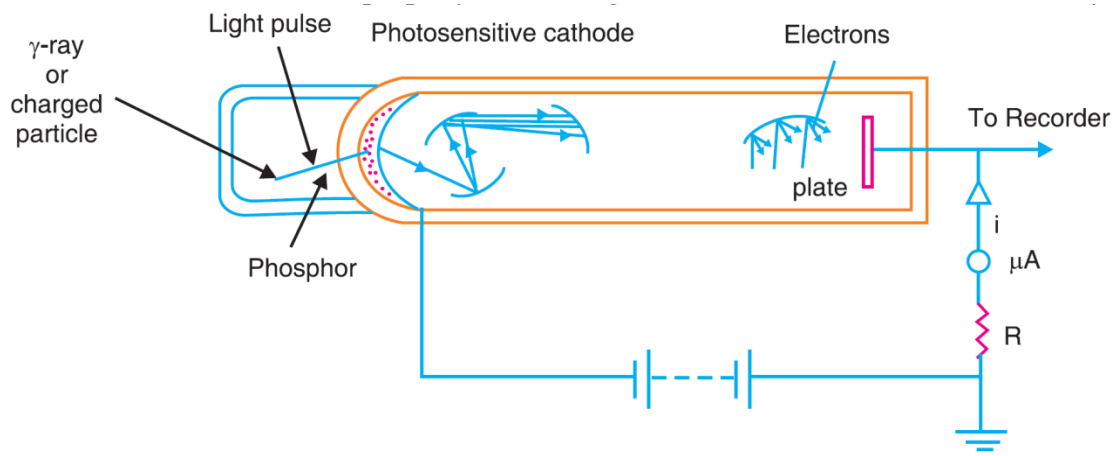


Fig 3.17

electron which strikes its surface. There may be from 10 to 14 such multiplier stages in a given tube. Hence from the emission of one single electron from the cathode, a burst of one million electrons may impinge on th final stage in the tube (the anode). The output pulse from the photomultiplier is fed to a pulse amplifie followed by a scaler circuit.

Solid-state Detectors

A *p-n* junction, used as a particle detector, is shown in Fig.3.19. It consists of a *p-n* junction between *p*-typ and *n*-type silicon. Contact is made with the *n*-type silicon layer by a thin evaporated film of gold. In order t minimise the current flowing in the detector, when no radiation is striking it, a reverse biased diode is alway used. The positive (reverse) bias applied to the gold film will push all the positive charge carriers away fror the junction and produce a *depletion layer*, indicated in the figure. The depletion layer contains almost n carriers of either sign. When an energetic charged particle travels through the depletion layer, its interactio with the electrons in the crystal, produces electron-hole pairs.

There is an electron-hole pair for every 3.5 *eV* (in *Si*) of energy lost by the charged particle. The electron and holes are swept away by the applied electric field and registered as a voltage pulse over the resistor *R*. Th number of charge carrier pairs produced in a semiconductor material is approximately 10 times as large as th number of ion pairs produced in a gas ion chamber *i.e*., the energy extended per pair is about 3.5 *eV* in silicor compared to about 30 *eV* for gases. The voltage pulse will therefore be about 10 times larger. Hence thi detector has much better *energy resolution* than other radiation detectors. In solid state detectors for charge particles, silicon has been used most because of its low intrinsic conductivity. This means that the detector ca be operated at room temperature without excessive leakage current. For gamma ray work, germaniur detectors are much better than silicon because of the larger density of germanium.

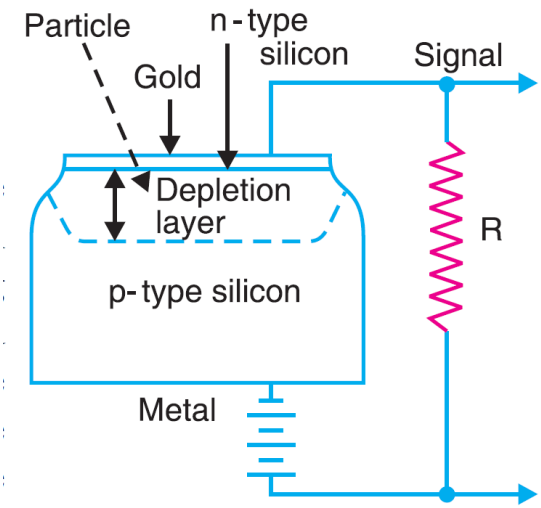
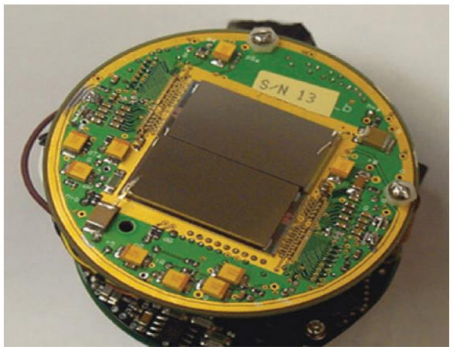


Fig 3.19



Solid-State Detector.

UNIT – IV – NUCLEAR REACTIONS AND NUCLEAR REACTORS

Nuclear Reactions

Nuclear reactions are processes in which one or more nuclides are produced from the collisions between two atomic nuclei or one atomic nucleus and a subatomic particle. The nuclides produced from nuclear reactions are different from the reacting nuclei (commonly referred to as the parent nuclei).

Following are some of the main types of nuclear reactions.

(i) Elastic scattering. In this case the incident particle strikes the target nucleus and leaves without loss of energy, but its direction may change.

Example : Scattering of α -particles from a thin gold foil.

$_{79}Au^{197} + {}_{2}He^{4} \rightarrow {}_{79}Au^{197} + {}_{2}He^{4}$

The target nucleus remains unaffected.

(ii) Inelastic scattering. In this case, the incident particle loses a part of its energy in exciting the target nucleus to a higher allowed energy level. The excited nucleus later decays to the ground

state, radiating the excess energy in the form of a γ -ray photon.

Example : $_{3}Li^{7} + {}_{1}H^{1} \rightarrow {}_{3}Li^{7*} + {}_{1}H^{1}$

$3Li7^{*} \rightarrow 3Li7 + g.$

(iii) Radiative Capture. Here the incident particle is captured by the target nucleus and a new nucleus is formed. The new nucleus, in general, has a considerable excess of energy and decays with the emission of one or more γ -ray photons.

Example : $_{6}C^{12} + {}_{1}H^{1} \rightarrow {}_{7}N^{13*} \rightarrow {}_{7}N^{13} + g$

(iv) Disintegration. Here the incident particle is absorbed by the target nucleus and the ejected particle is a different one. The composition of the resultant nucleus is also different from the parent nucleus. An example is the disintegration of beryllium by α -particle producing neutrons.

$_{4}Be^{9} + {}_{2}He^{4} \rightarrow {}_{6}C^{12} + {}_{0}n^{1}$

(v) Photodisintegration. When target materials are bombarded with radiations, the resulting compound nuclei are usually formed in excited states. These nuclei generally get rid of the excess excitation energy through neutron emission. For example,

$_{1}H^{2} + g \rightarrow {}_{1}H^{1} + {}_{0}n^{1}$

This requires a photon of energy 2.225 MeV.

Conservation Laws. The conservation laws may be stated and illustrated by reference to some specific nuclear reaction, say,

$_{5}B^{10} + {}^{2}He^{4} \rightarrow ({}_{7}N^{14}) \rightarrow {}_{1}H^{1} + {}_{6}C^{13} + Q$

(i) Conservation of charge : Total charge is conserved in every type of nuclear reaction. In B10 (α , p) C^{13} there are seven protons initially, also seven in the compound nucleus and in the products of the reaction.

(ii) Conservation of nucleons : The total number of nucleons entering and leaving the reaction is constant. In B^{10} (α , p) C^{13}, we find 14 nucleons at each stage of the reaction.

(iii) Conservation of mass-energy : In nuclear reactions neither kinetic energy nor rest mass is conserved by itself. But their total is always conserved.

(iv) Conservation of parity : The net parity before the reaction must equal the net parity after the reaction.

(v) Linear momentum, angular momentum, spin and isotopic spin are the other physical quantities which are also conserved in a nuclear reaction.

(vi) Quantities not conserved : The most prominent physical characteristics which are not conserved in nuclear reactions are the magnetic dipole moments and the electric quadrupole moments of the reacting nuclei. These moments depend upon the internal distribution of mass, charge and current within the nuclei involved and are not subject to conservation laws.

Conservation Laws. The conservation laws may be stated and illustrated by reference to some specific nuclear reaction, say,

$${}_5B^{10} + {}_2He^4 \rightarrow ({}_7N^{14}) \rightarrow {}_1H^1 + {}_6C^{13} + Q$$

(i) Conservation of charge : Total charge is conserved in every type of nuclear reaction.

In B^{10} (α , p) C^{13} there are seven protons initially, also seven in the compound nucleus and in the products of the reaction.

(ii) Conservation of nucleons : The total number of nucleons entering and leaving the reaction is constant. In B^{10} (α , p) C^{13}, we find 14 nucleons at each stage of the reaction.

(iii) Conservation of mass-energy : In nuclear reactions neither kinetic energy nor rest mass is conserved by itself. But their total is always conserved.

(iv) Conservation of parity : The net parity before the reaction must equal the net parity after the reaction.

(v) Linear momentum, angular momentum, spin and isotopic spin are the other physical quantities which are also conserved in a nuclear reaction.

(vi) Quantities not conserved : The most prominent physical characteristics which are not conserved in nuclear reactions are the magnetic dipole moments and the electric quadrupole moments of the reacting nuclei. These moments depend upon the internal distribution of mass, charge and current within the nuclei involved and are not subject to conservation laws.

Threshold Energy of an Endoergic Reaction

In an endoergic reaction $-Q$ is the energy which is needed to excite the reaction. This energy is supplied by K.E. of the incoming particle. But all the K.E. of the projectile particle is not available for the nuclear reaction because part of the energy is used to give K.E. to the compound nucleus. Hence, for a

reaction in which $-Q$ energy is to be absorbed, the incident particle should supply some energy in addition to $-Q$. Thus the minimum K.E. which the projectile should possess so that the nuclear reactions may take place is called the *threshold energy*.

Let m_i and v_i be the mass and initial velocity of the projectile. Let m_c and v_c be the mass and velocity of the compound nucleus. Applying conservation of momentum principle,

$$m_i v_i = m_c v_c \text{ or } v_c = m_i v_i / m_c.$$

$$\text{K.E. of compound nucleus} = \frac{1}{2} m_c v_c^2 = \frac{1}{2} m_c \left(\frac{m_i}{m_c}\right)^2 v_i^2$$

$$\text{Energy available for the reaction} = \frac{1}{2} m_i v_i^2 - \frac{1}{2} m_c v_c^2 = \frac{1}{2} m_i v_i^2 \left(1 - \frac{m_i}{m_c}\right)$$

But $m_c = m_i + m_t$ where m_t = mass of target nucleus.

$$\therefore \quad \frac{1}{2} m_i v_i^2 - \frac{1}{2} m_c v_c^2 = \frac{1}{2} m_i v_i^2 \left(\frac{m_t}{m_i + m_t}\right)$$

$$\text{or} \quad -Q = \frac{1}{2} m_i v_i^2 \left(\frac{m_t}{m_i + m_t}\right)$$

$$\therefore \quad \text{The threshold energy} = E_{th} = \frac{1}{2} m_i v_i^2 = -Q \left(\frac{m_i + m_t}{m_t}\right)$$

Example. *The Q value of the* Na^{23} (n, α) F^{20} *reaction is –5.4 MeV. Determine the threshold energy of the neutrons for this reaction.*

SOL. Here, $Q = -5.4$ MeV; $m_i = 1.008665u$, $m_t = 22.9898\ u$.

$$E_{th} = -Q \left(\frac{m_i + m_t}{m_t}\right) = -(-5.4)\left(\frac{1.008665 + 22.9898}{22.9898}\right)$$

$$= 5.635 \text{ MeV}.$$

Energy Balance in Nuclear Reactions and the Q-value

In all nuclear reactions, the total sum of mass and energy is conserved. Thus taking the equation A + B → P + O, the target is supposed to be at rest and let its mass be m1. The projectile has a mass m2 and K.E. = E_2.

The product nucleus has a mass m_3 *and K.E.* E_3. *The outgoing particle has a mass* m_4 *and K.E.* E_4. *The equation representing the conservation of total energy is written as*

$$m_1 c^2 + m_2 c^2 + E_2 = m_3 c^2 + E_3 + m_4 c^2 + E_4$$

$$Q = E_3 + E_4 - E_2 = (m_1 + m_2 - m_3 - m_4) c^2.$$

The quantity Q is called the energy balance or the Q-value of the reaction. If Q is positive, the reaction is exothermic (or exoergic). Now the K.E. of products of transmutation is greater

than the K.E. of the reactants and energy is released in the process. If Q is negative, the reaction is endothermic (or endoergic). The equation for a nuclear reaction is written as

$$A + B \rightarrow P + O + Q$$

where Q may be positive or negative.

Example 1. *Consider the reaction* ${}_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1 + Q$

where Q denotes the energy absorbed or evolved during the nuclear reaction

Sol. The atomic masses of the particles are : $N^{14} = 14.003074$, $He^4 = 4.002604$, $O^{17} = 16.99913$ and $H^1 = 1.007825u$.

Substituting these values in the equation,

$$14.003074 + 4.002604 = 16.99913 + 1.007825 + Q$$

$$\therefore \quad Q = 18.005678 - 18.006955 = -0.001277u$$

According to Einstein's mass-energy relation, $1\ u = 931.3$ MeV

$\therefore$ The calculated value of $Q = -0.001277 \times 931.3 = -1.189$ MeV.

This is in close agreement with the experimental value (*i.e.*, –1.26 MeV). Incidentally, Einstein's mass-energy relation gets verified.

Example 2. *Determine the product nuclei and Q values in the following reactions :* Al^{27} *(d,* α*) and* Mg^{25} *(*α*, d). Masses of* Al^{27}*,* Mg^{25}*,* α *and d are 26.9901, 24.9936, 4.0039 and 2.0147 amu respectively. Comment on your results.*

Sol. (*i*) The nuclear reaction is

$${}_{13}Al^{27} + {}_1H^2 \rightarrow {}_{12}Mg^{25} + {}_2He^4 + Q$$

$$\therefore \quad Q = 26.9901 + 2.0147 - 24.9936 - 4.0039 = 0.0073 \text{ amu.}$$

$$i.e., \quad Q = 0.0073 \times 931.3 \text{ MeV} = 6.797 \text{ MeV.}$$

(*ii*) ${}_{12}Mg^{25} + {}_2He^4 \rightarrow {}_{13}Al^{27} + {}_1H^2 + Q$

$$\therefore \quad Q = 24.9936 + 4.0039 - 26.9901 - 2.0147 = -0.0073 \text{ amu.}$$

$$i.e., \quad Q = -0.0073 \times 931.3 \text{ MeV} = -6.797 \text{ MeV.}$$

The Scattering Cross-Section

The properties of scattering interactions are usually expressed in terms of the Scattering cross-section. To define this term, we consider the following scattering experiment : A beam of particles (called beam particles) is directed on a scatterer consisting of target particles. As a result of collisions between beam and target particles, there are particles which emerge from the reactions (called reaction products) and these are detected in particle detectors. To describe this quantitatively, we suppose that the scatterer contains N_t target particles. The target is uniformly illuminated by a flux F_s (expressed as the number of beam particles per unit area per unit time arriving at the target) of beam particles. We suppose also that the scatterer is sufficiently small that the beam is negligibly attenuated in passing through it. Then, if

there are δ Ns scattering interactions per unit time which lead to detected particles, we define the scattering cross section $\delta\sigma$ as $\delta\sigma = \delta$ Ns /Nt Fs. In the limit that the detectors subtend very small solid angles, as seen from the target, we define the differential scattering cross section $d\sigma$. When a single detector subtending a solid angle $\delta\Omega$ is used to define $\delta\sigma$, we may define the cross section per unit solid angle as

$$\frac{d\sigma}{d\Omega} = \lim_{\delta\Omega\to 0} \frac{\delta\sigma}{\delta\Omega}$$

The total scattering cross section σ is obtained by summing $\delta\sigma$ over all scattering events: $\sigma = \Sigma\,\delta\sigma$. It is customary practice to associate a cross-section with each particular type of nuclear interaction. A scattering cross-section is used when dealing with nuclear scattering processes; an absorption cross section for nuclear absorption processes; and a fission cross-section when studying nuclear collisions, leading to the fission of the target nucleus.

Determination of cross-section.

Consider a slab of some material whose area is *A* and thickness is *dx* (Fig.4.1). Its volume is *Adx*. If the target material contains *n* nuclei per unit volume, the total No. of nuclei in the slab = *nAdx*. Each nucleus has a cross-section of σ for some particular interaction, so that the aggregate crosssection of all the nuclei in the slab is $nA\,\sigma\,dx$. Let *N* be the No. of incident particles in a bombarding beam. Let *dN* be the No. of particles that interact with nuclei in the slab. Then,

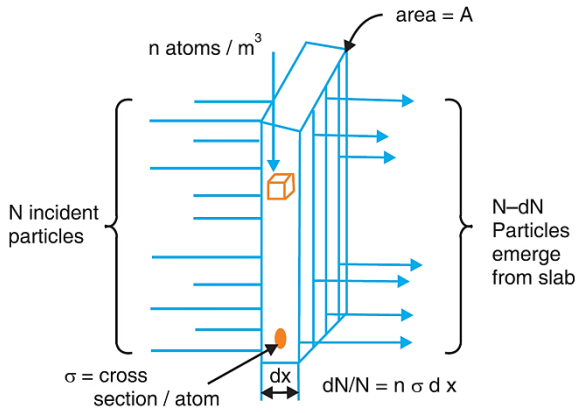


Fig.4.1

$$\frac{dN}{N} = \frac{\text{Aggregate cross section}}{\text{target area}} = \frac{nA\sigma dx}{A} = n\sigma dx.$$

Now let us consider the same beam of particles incident on a slab of thickness *x*. Let N_0 be the number of incident particles. If each particle can interact only once, *dN* particles may be thought of as being removed from the beam in passing through the first *dx* of the slab. Hence

$$\frac{-dN}{N} = n\sigma dx \quad \text{or} \quad \int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx$$

$$\text{or} \qquad \log_e N - \log_e N_0 = -n\sigma x$$

$$\text{or} \qquad N = N_0 e^{-n\sigma x}$$

The number of surviving particles *N* decreases exponentially with increasing slab thickness *x*. Here, σ is the cross-section per nucleus or the *microscopic cross-section. n* is the number of nuclei per unit volume. The product $n\sigma$ is called the *macroscopic cross-section and is denoted by* Σ. Σ *has the dimensions of* L^{-1}.

Thus $N = N_0 e^{-\Sigma x}$

The unit used for σ is an area of 10^{-28} m^2. This unit is called the *barn*.

If the incident flux (beam intensity) is *I*0 and the flux after penetrating a distance *x* is *I*, then $I = I_0 e^{-\Sigma x}$ or

$$\log_e I = \log_e I_0 - \Sigma x$$

Σ can be found by measuring the beam intensity with and without targets. A plot of $\log_e$ I against x for several foil thicknesses gives Σ graphically and hence σ .

The cross-sections for most nuclear reactions depend upon the energy of the incident particle.
Fig.4.2 shows how the neutron capture cross-section of ${}_{48}Cd^{113}$ varies with neutron energy. In
this reaction, the absorption of a neutron is followed by the emission of a gamma ray :
Cd^{113} (n, γ) Cd^{114}. The narrow peak at 0.176 eV is a resonance effect associated with an excited
state in the Cd^{114} nucleus. The capture cross-sections of Cd^{113} for slow neutrons are so great
that cadmium is widely used in control rods for nuclear reactors

Macroscopic cross-section and mean free path. The mean free path (λ) of a particle in a material is the average distance it can travel in the material
before interacting there.

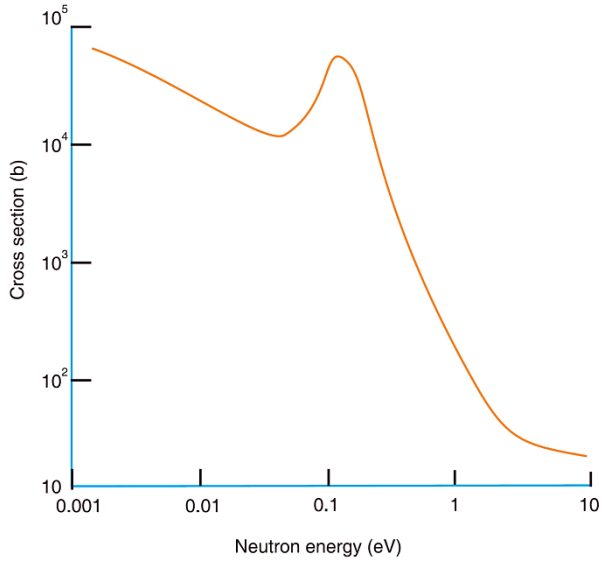


Fig 4.2

$$1 = \frac{\int_0^\infty x e^{-\Sigma x}\, dx}{\int_0^\infty e^{-\Sigma x}\, dx} = \frac{1}{\Sigma}$$

Example. The usefulness of cadmium in a nuclear reactor depends on the high thermal absorption cross section of the 113 isotope. viz., σ_α (Cd^{113}) = 21000 barns. If the density of cadmium is 8.7×10^3 kg m^{-3}, calculate the macroscopic cross-section of Cd^{113} and hence the thickness required to attenuate a neutron beam to 0.01% of its original intensity.

SOL. Macroscopic cross section = $\Sigma = \sigma N_v$

where N_v = Number of nuclei per unit volume in the absorber.

Now unit volume of material of density ρ contains $N_A\rho/A$ nuclei, where N_A is the Avogadro constant and A is the atomic mass number of the absorber atoms.

Thus $$N_v = \frac{N_A}{A}\rho \quad \text{and} \quad \Sigma = \sigma\frac{N_A}{A}\rho$$

$$\therefore \quad \Sigma = \frac{(6.02\times10^{26})(8.7\times10^{3})(21000\times10^{-28})}{113} = 97000\,\text{m}^{-1}$$

From $I = I_0 e^{-\Sigma x}$ we have

$$\Sigma x = \log_e\left(\frac{I_0}{I}\right) = 2.3\log_{10}10000 = 2.3\times4 = 9.2$$

$$\therefore \quad x = \frac{9.2}{97000}\text{m} = 95\times10^{-6}\,\text{m}.$$

Nuclear Fission

The process of breaking up of the nucleus of a heavy atom into two, more or less equal fragments with the release of a large amount of energy is known as fission.

When uranium is bombarded with neutrons, a uranium nucleus captures a slow neutron, forming an unstable compound nucleus. The compound nucleus splits into two nearly equal parts. Some neutrons are also released in this process.

The schematic equation for the fission process is

The schematic equation for the fission process is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow X + Y + neutrons \qquad ...(1)$$

$_{92}U^{236*}$ is a highly unstable isotope, and X and Y are the fission fragments. The fragment are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are given off. Typical fission reactions are

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q \qquad ...(2)$$

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + Q \qquad ...(3)$$

where Q is the energy released in the reaction.

According to Eqn. (2), when $_{92}U^{235}$ is bombarded by a slow moving neutron, the nucleus b unstable ($_{92}U^{236*}$) and splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons and energy Q (Fig. ...).

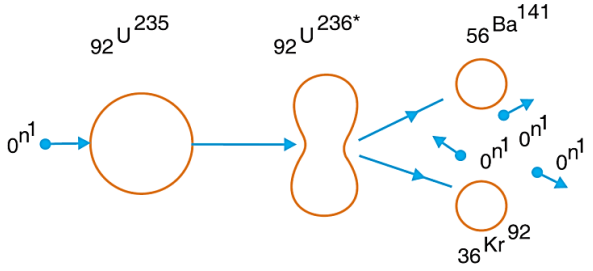
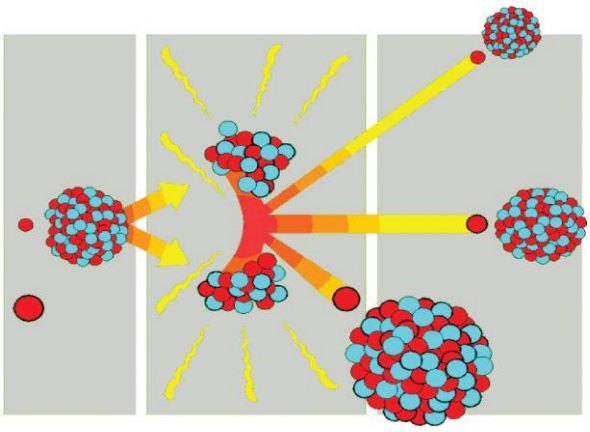


Fig 4.3

Chain Reaction

A chain reaction is a self-propagating process in which number of neutrons goes on multiplying rapidly almost in geometrical progression during fission till whole of fissile material is disintegrated.

Example : Suppose a single neutron causing fission in a uranium nucleus produces 3 prompt neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing 9 neutrons. These nine neutrons in turn may cause fission in nine uranium nuclei producing 27 neutrons and so on. The number of neutrons produced in n such generations is 3^n. The ratio of secondary neutrons produced to the original neutrons is called the *multiplication factor* (k).



Chain reaction

Consider 1 kg of $_{92}U^{235}$ which contains $6.023 \times 10^{26}/ 235$ or about 25×10^{23} atoms. Suppose a stray neutron causes fission in a uranium nucleus. Each fission will release on the average 2.5 neutrons. The velocity of a neutron among the uranium atoms is such that a fission capture of a thermal neutron by the $_{92}U^{235}$ nuclei takes place in about 10^{-8} s. Each of these fissions, in turn, will release 2.5 neutrons. Let us assume that all these neutrons are available for inducing further fission reactions. Let n be the number of stages of fission captures required to disrupt the entire mass of 1 kg of $_{92}U^{235}$. Then

$$(2.5)^n = 25 \times 10^{23} \text{ or } n \approx 60.$$

The time required for 60 fissions to take place $= 60 \times 10^{-8}\ s = 0.6\ \mu\ s$.

Since each fission releases about 200 MeV of energy, this means that a total of $200 \times 25 \times 10^{23} = 5 \times 10^{26}$ MeV of energy is released in 0.6 μ s. The release of this tremendous amount of energy in such a short time interval leads to a violent explosion. This results in powerful air blasts and high temperature of the order of 10^7 K or more, besides intense radioactivity. The self-propagating process described here is called a *chain reaction.*

Two types of chain reaction are possible. In one, the chain reaction is first accelerated so that the neutrons are built up to a certain level and there after the number of fission producing neutrons is kept constant. This is *controlled chain reaction*. Such a controlled chain reaction is used in nuclear reactors. In the other type of chain reaction, the number of neutrons is allowed to multiply indefinitely and the entire energy is released all at once. This type of reaction takes place in atom bombs.

Multiplication factor (k). The ratio of secondary neutrons produced to the original neutrons is called the multiplication factor. It is defined as

$$k = \frac{\text{Number of neutrons in any one generation}}{\text{Number of neutrons in the preceding generation}}.$$

The fission chain reaction will be "*critical*" or steady when $k = 1$, it will be building up or "*Supercritical*" when $k > 1$ and it will be dying down or "*subcritical*" when $k < 1$.

Critical size for maintenance of chain reaction. Consider a system consisting of uranium (as fissile material) and a moderator. Even though each neutron that produces fission ejects 2.5 neutrons on an average, all of them are not available for further fission. The maintenance of the chain reaction depends upon a favourable balance of neutrons among the three processes given below :

(1) The fission of uranium nuclei which produces more neutrons than the number of neutrons used for inducing fission.

(2) Non-fission processes, including the radiative capture of neutrons by the uranium and the parasitic capture by the different substances in the system and by impurities.

(3) Escape or leakage of neutrons through the surface of the system. If the loss of neutrons due to the last two causes is less than the surplus of neutrons produced in the first, a chain reaction takes places. Otherwise it cannot take place. The escape of neutrons takes place from the surface of the reacting body and fission occurs throughout its *volume.*

∴ Escape rate varies as r^2 and production rate varies as r^3.

$$\frac{\text{Escape rate}}{\text{Production rate}} \propto \frac{1}{r}.$$

The larger the size of the body, the smaller is the escape rate. Thus it is clear that by increasing the volume of the system, the loss of neutrons by escape from the system is reduced. The greater the size of the system, the lesser is the probability of the escape of neutrons. In this case, the production of neutrons will be more than the loss due to other causes and a chain reaction can be maintained. Thus there is a critical size for the system. *Critical size of a system containing fissile material is defined as the minimum size for which the number of neutrons produced in the fission process just balance those lost by leakage and non-fission capture.*
The mass of the fissionable material at this size is called the *critical mass*. If the size is less than the critical size, a chain reaction is not possible.

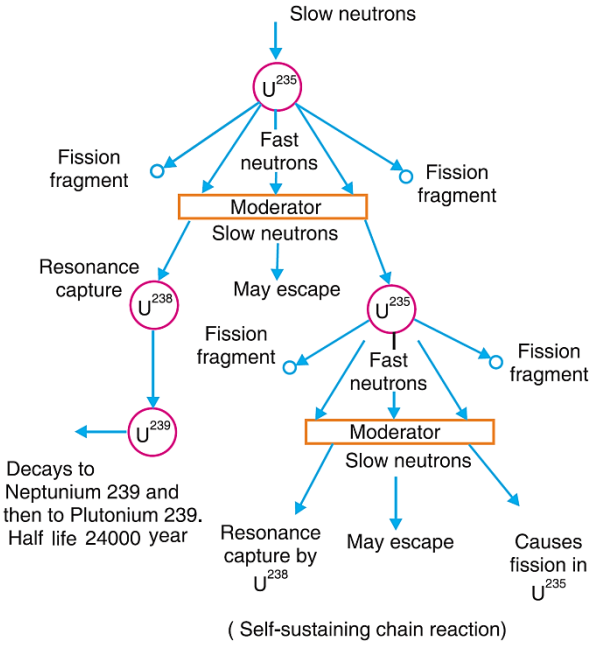


Fig 4.4

Natural uranium and chain reaction.

Natural uranium consists of 99.28% of U^{238} and 0.72% of U^{235}. As most of the mass of natural uranium consists of U^{238}, the neutrons released during nuclear fission will try to bombard the nuclei of U^{238} mostly and very few will bombard U^{235}. U^{235} undergoes fission even by neutrons of small energy like thermal neutrons.

U^{238} is fissionable only with fast neutrons of energy 1 MeV or more. It has been found that very few neutrons can cause fission of U^{238} but neutrons of all possible energies can cause fission of U^{235}. Thus chain reaction is not possible in natural uranium. A chain reaction can, however, be made to develop in natural uranium, if the fast neutrons from it are quickly reduced to thermal ones before they are lost through non-fission capture in the uranium, so that the chances of the thermal neutron fission of U^{235} go up. The neutron can be slowed down by distributing among lumps or rods of uranium a material called *moderator*. The moderators must not absorb the neutrons. The function of the moderator is to slow down the neutrons produced by fission by elastic collision. Materials used as neutron moderators have a large inelastic scattering cross section and, at the same time, a small neutron-capture cross-section. Commonly used moderators are graphite, heavy water (D_2O), beryllium, beryllium oxide, hydrides of metals and organic liquids. The nuclei of these substances absorb neutrons only to a slight extent.

Fig. 4.4 shows a self-sustaining chain reaction. A slow neutron bombards a U^{235} nucleus. The nucleus breaks into two fragments and in the fission process three fast neutrons are emitted. The neutrons are slowed down by the moderator. One neutron may escape. One neutron may be captured by U^{238} to form U^{239} which decays to Np^{239}, and then to Pu^{239}. One neutron is still available for carrying on the chain reaction. It bombards U^{235} and the process is repeated.

Nuclear Reactors

During the fission of $U235$ a large amount of energy is released. The atom bomb is due to an uncontrolled chain reaction. A very large amount of energy is liberated within an extremely small interval of time. Hence it is not possible to direct this energy for any useful purpose. But in a nuclear reactor, the chain reaction is brought about under controlled conditions. If the chain reaction is put under control, after some time a steady state is established. Under a steady state, the rate of energy production also attains a constant level. Such a device in which energy is released at a given rate is known as a *nuclear reactor.*

Nuclear reactor

Nuclear reactors consist of five main elements:

(1) The fissionable material called fuel,

(2) Moderator,

(3) Neutron reflector,

(4) Cooling system, and

(5) The safety and control systems.

(1) The fissionable substance. The commonly used fissionable materials are the uranium isotopes U^{233}, U^{238}, the thorium isotope Th^{232}, and the plutonium isotopes Pu^{239}, Pu^{240} and Pu^{241}.

(2) Moderator. The function of the moderator is to slow down the highly energetic neutrons produced in the process of fission of U^{235} to thermal energies. Heavy water (D_2O), graphite, beryllium, etc., are used as moderators. Ideally, moderators have low atomic weight and low absorption cross section for neutrons.

(3) Neutron reflector. By the use of reflectors on the surface of reactors, leakage of neutrons can be very much reduced and the neutron flux in the interior can be increased. Materials of high scattering cross-section and low absorption cross-section are good reflectors.

(4) Cooling system. The cooling system removes the heat evolved in the reactor core. This heat is evolved from the K.E., of the fission fragments when they are slowed down in the fissionable substance and moderator. The coolant or heat-transfer agent (water, steam, *He*, CO_2, air and certain molten metals and alloys) is pumped through the reactor core. Then, through a heat exchanger, the coolant transfers heat to the secondary thermal system of the reactor.

(**5) Control and safety system.** The control systems enable the chain reaction to be controlled and prevent it from spontaneously running away. This is accomplished by pushing control rods into the reactor core. These rods are of a material (boron or cadmium) having a large neutron-absorption cross-section. These rods absorb the neutrons and hence cut down the reactivity. By pushing in the rods, the operation of the reactor can be made to *die down*, by pulling them out to *build up*. The safety systems protect the space surrounding the reactor against intensive neutron flux and gamma rays existing in the reactor core. This is achieved by surrounding the reactor with massive walls of concrete and lead which would absorb neutrons and gamma rays.

Power reactor. The heat generated in a nuclear reactor is used for producing power in a nuclear power plant. Fig. 4.5 shows the components of a power reactor. A quantity of encriched uranium in the form of pure metal or solution of a soluble salt in water constitutes the centre of the heat energy source. A large quantity of heat is produced in the fission process. The cadmium rods regulate the temperature, to a pre-determined value. If it is desired to bring down the temperature, the cadmium rods are pushed down further as to absorb more neutrons. If the temperature has to be raised, the cadmium rods are pulled up a little. A fluid is circulated through the shielded reactor and heat exchanger. The hot fluid, while flowing through the heat exchanger, converts water into steam. The steam produced runs conventional turbines to produce electricity.

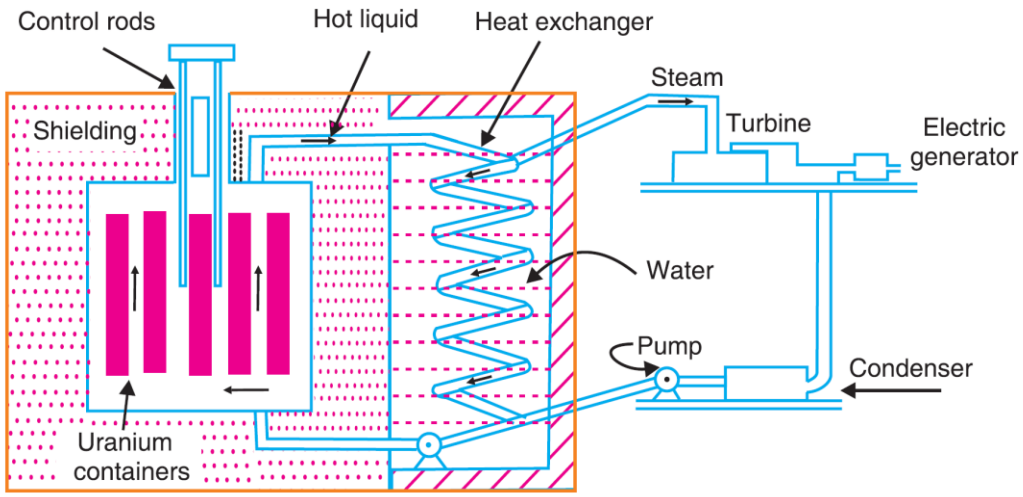


Fig 4.5

Breeder Reactor. If a thermal reactor core with U^{235} fuel is surrounded by a blanket of a fertile material like U^{238}, U^{238} can be converted into fissile fuel. Reactors of this type are called fuel producing reactors. The reactions are as follows :

$$_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} + \gamma$$

This is followed by $\quad {}_{92}U^{239} \rightarrow {}_{93}Np^{239} + \beta^-$

${}_{93}Np^{239}$ is also radioactive. It emits a β-particle to form plutonium.

$$_{93}Np^{239} \rightarrow {}_{94}Pu^{239} + \beta^- + \gamma\,.$$

This process of producing one type of fissionable material (Pu^{239}) from a non-fissionable material (U^{238}) is called *breeding* and the reactor a *breeder reactor.*

The breeding reactions for the fertile material ${}^{232}_{90}Th$ are

$${}^{232}_{90}Th + n \rightarrow {}^{233}_{90}Th^* \xrightarrow[22\,\text{min}]{\beta^-} {}^{233}_{91}Pa \xrightarrow[27\,\text{d}]{\beta^-} {}^{233}_{92}U\ (T_{1/2} = 1.6\times10^5\ \text{yrs})\,.$$

Uses of Nuclear Reactors.

(1) Nuclear power. Nuclear reactors are used in the production of electric energy.

(2) Production of radioisotopes. Nuclear reactors are useful in producing a large number of radio-isotopes. To produce radio-isotope, a suitable compound is drawn into the centre of the reactor core where the flux of neutrons may well be more than 10^{16} /m^2/ sec. Sodium-24 is manufactured in this way.

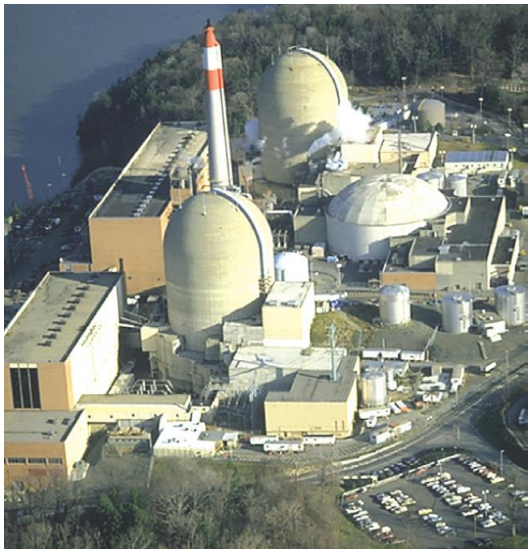
$$11Na^{23} + {}_0n^1 \rightarrow 11Na^{24*}$$

(3) Scientific research. Reactors produce a number of radioactive materials needed for research purposes. The reactors provide a huge source of neutrons. Using these neutrons, several useful radioisotopes have been artificially produced and several nuclear reactions have been studied. We may

also study the effect of neutrons on biological tissues. Reactors may also be used to study radiation damage.

Pressurized Water Reactor (PWR)

If the pressure on the water surface is increased, its boiling point increases so that it can have more quantity of heat energy per unit mass. This principle is utilised in PWR. Fig. 4.6 shows a simple sketch of a PWR. The fuel is in the form of uranium oxide (UO_2). It is sealed in long, thin zirconium-alloy tubes that are assembled together with movable control rods into a core. The core is enclosed in a steel pressure vessel. Light water is used as coolant and moderator.



Pressurised water nuclear reactor

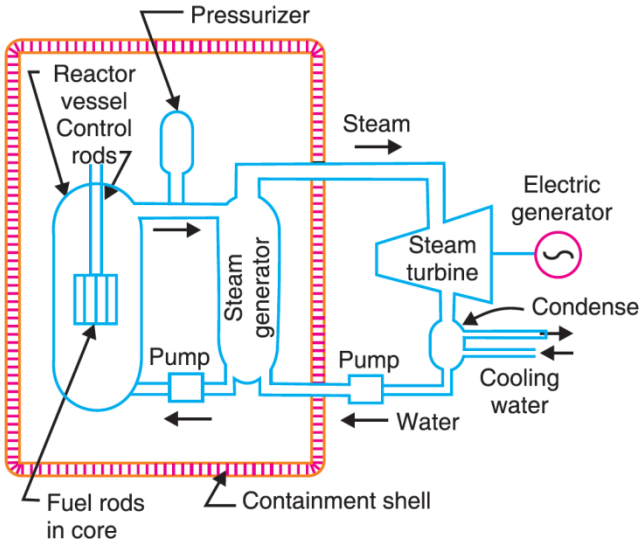


Fig 4.6

The water that circulates past the core is kept at a sufficiently high pressure, about 150 atm, to prevent boiling. The water enters the pressure vessel at about 280º C and leaves at about 320º C. The water then passes through a heat exchanger (steam generator) which produces steam. The resulting steam then passes out of the containment shell. The containment shell serves as a barrier to protect the outside world from accidents to the reactor. The high pressure steam drives a turbine. The turbine drives an electric generator and produces electrical power. Steam after running the turbine gets condensed into water in the condenser. This water is again circulated through heat exchanger by means of a pump.

Boiling Water Reactor (BWR)

Uranium is used as fuel in BWR (Fig. 4.7). Water is used as a coolant and moderator. Circulating water absorbs heat produced by fission of the fuel and gets converted into high pressure steam in the reactor core itself. This steam runs the turbine which in turn produces electric power. Steam after running the turbine gets condensed into water in the condenser. This water is again circulated into the reactor core by using a pump. The steam leaving the reactor may be slightly radioactive. Therefore, the pipe lines and turbine assembly are properly shielded.

Boiling water reactor

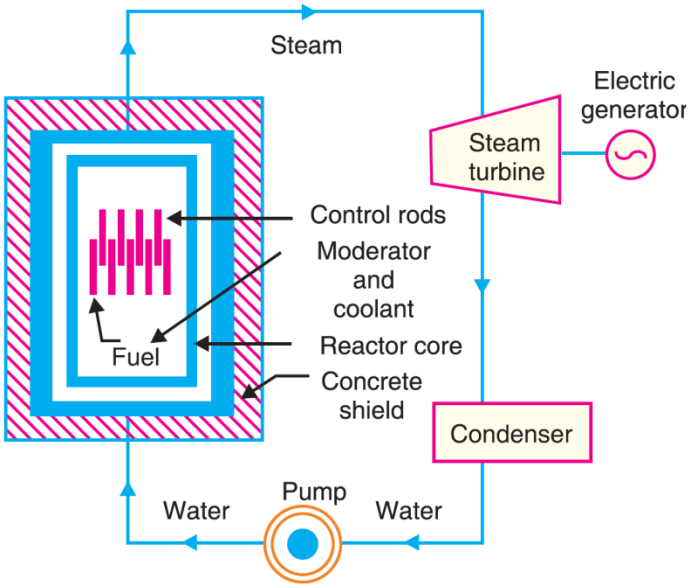
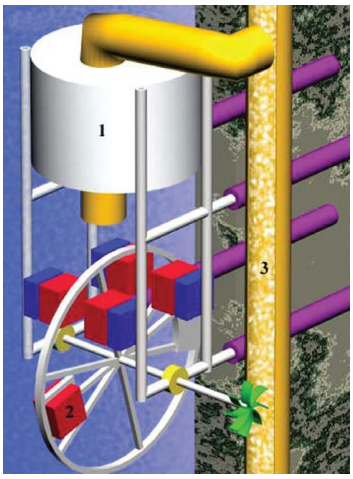


Fig 4.7

Fast Breeder Reactor

It is possible to run a reactor on fast neutrons by using a fuel with $^{239}_{94}Pu$. The fast neutrons released in the fission are used to transform $^{238}_{94}U$ into $^{239}_{92}Pu$. A fast breeder reactor is one designed to function with fissions induced by fast neutrons. The core of such a reactor will contain two materials, say, fissionable fuel $^{239}_{94}Pu$ and the potential fuel $^{238}_{92}U$. Existing breeder reactors are all of the fast type. They use a mixture of UO_2 and PuO_2 as fuel. The coolant is liquid sodium. Fig. 4.8 shows a liquid-sodium-cooled fast breeder reactor. The core may consist of ^{235}U and ^{239}Pu, while the blanket contains the fertile ^{238}U that will breed into fissionable material. Liquid



Fast breeder reactor

sodium is very effective as a medium for heat transfer. But its chemical reactivity and the radioactivity induced in it by neutron irradiation pose serious problems. To prevent contaminating the steam-generating system in the event of a sodium leak, an intermediate liquid sodium cooling loopis used between the primary loop that passes through the reactor core and the steam generator.

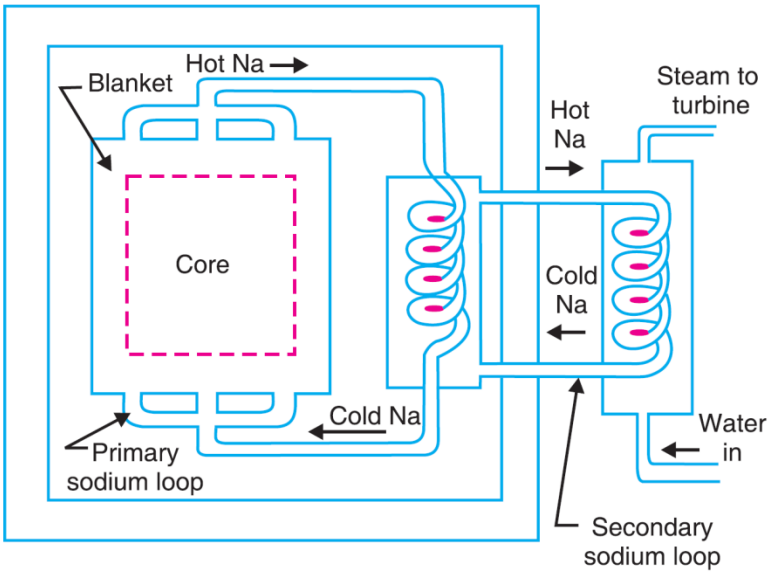


Fig 4.8

Neutron Cycle in a Thermal Nuclear Reactor

Let us study the behaviour of neutrons in a reactor assembly consisting of enriched uranium (a mixture of the uranium isotopes $^{235}_{92}U$ and $^{238}_{92}U$) and a moderator. For simplicity we assume that the reactor is infinitely large so that there is no leakage of neutrons through its surface. Let us start with the fission of a $^{235}_{92}U$ nucleus by a thermal neutron. In this fission, the number of fast neutrons produced is ν. Of these, some will cause a fission in $^{238}_{92}U$, and therefore create additional fast neutrons. We take account of this small increase in the neutrons by introducing a factor, namely, ε, called the *fast fission factor*. The number of fast neutrons available is now $\nu\ \varepsilon$, where $\varepsilon > 1$. ε is usually about 1.03.

These $\nu\ \varepsilon$ neutrons diffuse through the pile and are slowed down by collisions with moderator nuclei. However, a few of them are captured by $^{238}_{92}U$ before they are slowed down to thermal energies. This is known as resonance absorption. Therefore, we define *resonance escape probability*, *p*, as the fraction of neutrons escaping resonance absorption. Hence, the number of neutrons surviving thus far and reaching thermal energies is $\nu\ \varepsilon\ p$. *p* is usually 0.95. Of these $\nu\ \varepsilon\ p$ thermal neutrons, a fraction *f* called the *thermal utilisation factor* is absorbed in $^{235}_{92}U$ nuclei while the rest are absorbed in moderator and other structural materials. *f* is always less than 1. Thus the number of 235 92U nuclei undergoing fission is $\nu\ \varepsilon\ p\ f$. When these neutrons are absorbed by $^{235}_{92}U$ in the fuel, some cause fission, whereas some produce other kinds of reactions such as capture. The fraction of neutrons causing fission is given by the ratio of fission cross-section (σf) to the total absorption cross-section (σa).

$\therefore$ the number of second generation thermal neutrons causing fission of $^{235}_{92}U = \nu\varepsilon p f \dfrac{\sigma_f}{\sigma_a}$.

Now, $\nu\dfrac{\sigma_f}{\sigma_a} = \eta$ = *the average number of fission neutrons released per neutron absorbed by a fissionable nuclide.*

The ratio *K* of the number of fissions produced to a single neutron is

$$K = \eta\ \varepsilon\ pf.$$

This is the *multiplication factor*. We have assumed that the reactor assembly is of an infinite size so that there is no leakage of neutrons. Hence

$$K_\infty = \eta \ \varepsilon \ pf.$$

This relation is called the *four-factor formula*. In designing a nuclear reactor, the fundamental problem is to maintain *K* at 1 in order to achieve a self-sustaining chain reaction. Of the four factors, η and ε are fixed for a given fuel. The other two factors *p* and *f* are made as large as possible by properly designing the reactor geometry, arranging the fuel, and choosing the moderator.

Fusion Reactors

In the experiments conducted by bombarding the particles $^{2}{}_{1}H$ and $^{3}{}_{1}H$ with the deuterons, the following reactions have been found to occur:

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + 4.0\,MeV \quad ...(1)$$

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n + 3.3\,MeV \quad ...(2)$$

$$^{3}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.6\,MeV \quad ...(3)$$

Each of the deuteron-deuteron reactions (1) and (2) has a small cross-section. However, the deuteron triton reaction, (3) is much more probable and also liberates a large amount of energy. The first fusion reactors are more likely to employ a deuterium—tritium mixture. Three conditions must be met by a successful fusion reactor.

1. We know that for a fusion reaction to take place, the colliding particles should have enough energies to overcome the coulombian repulsive forces between them. This needs energies of the order of 0.1 MeV which corresponds to a temperature of approximately 10^8 K. This means that for a fusion reaction to take place temperatures of this order should prevail initially. At such high temperatures, the atoms are ionized into positively charged ions (*i.e.*, nuclei) and electrons. This completely ionised and electrically neutral state of matter is called *plasma.*

2. There should be a fairly high density of the nuclei to ensure that collisions between nuclei are frequent. An important parameter for the fusion reaction is the *reaction rate* which can be represented as

$$R12 = n1n2 < \sigma v12 > \text{reactions/m}^2\text{/sec.}$$

Here *n*1, *n*2 represent particle densities of the colliding particles, v_{12}, the relative velocity of the particles and $< \sigma v12 >$ represents the average of the product of cross-section σ and the relative velocity of the particles. If the colliding particles are identical, we have

$$R_{11} = \frac{1}{2} n^2 < \sigma v > \text{reactions/m}^2\text{/sec.}$$

3. The assembly of reacting nuclei must remain together for a long enough time to give off more energy than the reactor's operation takes.

It is difficult to achieve a high enough particle density *n* for a long enough time τ in a hot plasma to obtain a net energy yield. According to the *Lawson criterion*, the product of the density and the confinement time (*i.e.*, the product $n\tau$) of the plasma should be about 2×1022 particles sm–3 and 1020 particles sm–3 for the deuteron-deuteron and the deuteron-triton fusion reactions respectively to occur in it.

Plasma Confinement

(*i*) Magnetic confinement. For the confinement of the plasma, one cannot use the walls of any vessel. Any contact with the wall will not only quickly cool the plasma but also cause the wall to evaporate. There are at present two schemes under investigation for confining the thermonuclear fuel: (*i*) *magnetic confinement* and (*ii*) *inertial confinement*. In magnetic confinement, the plasma is confined by a carefully designed magnetic field. Two classes of field geometry are being studied. They are: (1) Magnetic bottle and (2) Tokamak.

1. Magnetic bottle. It is based on the reflection of an ion that moves in a magnetic field *B* whose lines of force converge. The magnetic force on an ion of velocity *v* is perpendicular to both *v* and *B*. So the force has a backward component in such a field as well as the inward component that causes the ion to follow a helical path around the lines of force (Fig. 4.9). If the backward force is powerful enough, the converging field acts as a mirror to reverse the ion's direction of motion. Thus, we can form a high density of magnetic field lines which reflects the particles back into the low-field region and is hence known as *a magnetic mirror*. A pair of magnetic mirrors constitutes a magnetic bottle (Fig. 4.10).

2. Tokamak. A 'tokamak' is a magnetic bottle in which the confining field is created by a combination of currents flowing in external coils and currents flowing in the plasma. It is a toroidal magnetic trap. The principle of tokamak method of magnetic confinement is shown in Fig. 4.11. A toroidal field is produced by a winding of coils, and a poloidal field is produced by an axial current. These two fields are combined in the tokamak design. (The current-carrying coils are not shown). The resulting field lines form a helix, through which the ions can travel in closed orbits.

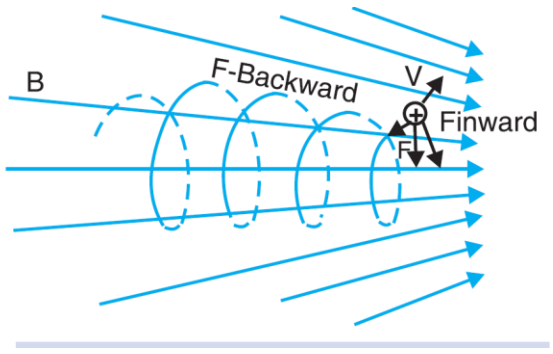


Fig 4.9

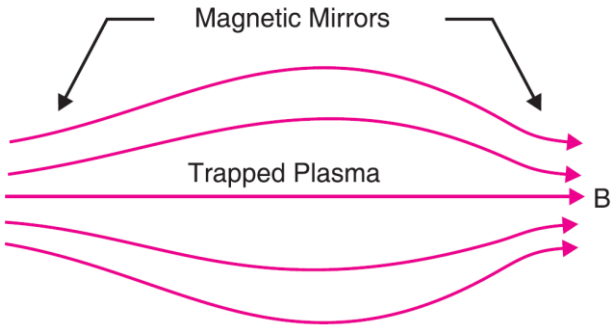


Fig 4.10

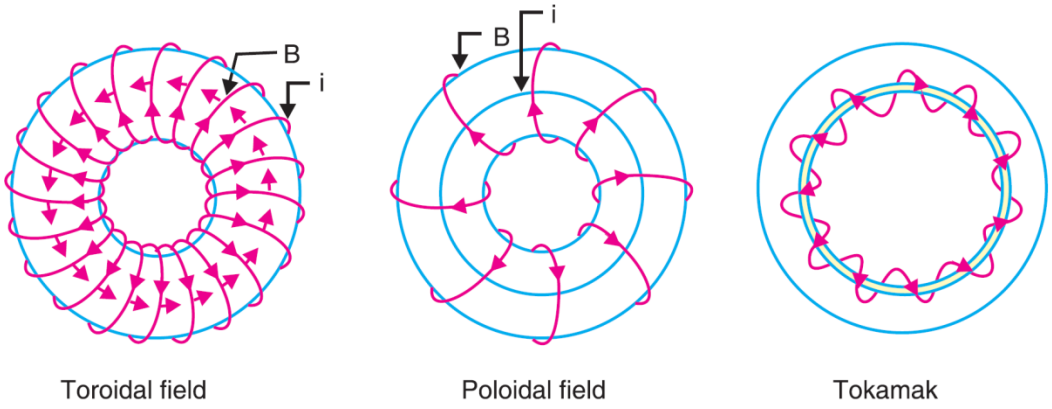


Fig 4.11

(*ii*) Inertial confinement. This scheme employs energetic beams to both heat and compress tiny deuterium-tritium pellets. Here, the fusion fuel (*e.g.*, mixture of deuterium and tritium), in the form of a pellet, is imploded from all sides by energy sources such as laser beams, high energy electron or ion beams (Fig. 4.12). The intense compression pressures and the high temperatures produced in the pellet may produce conditions conducive to fusion. The difficulties in this approach are the low efficiencies of laser or other sources, and the need to produce stable symmetrical implosion.

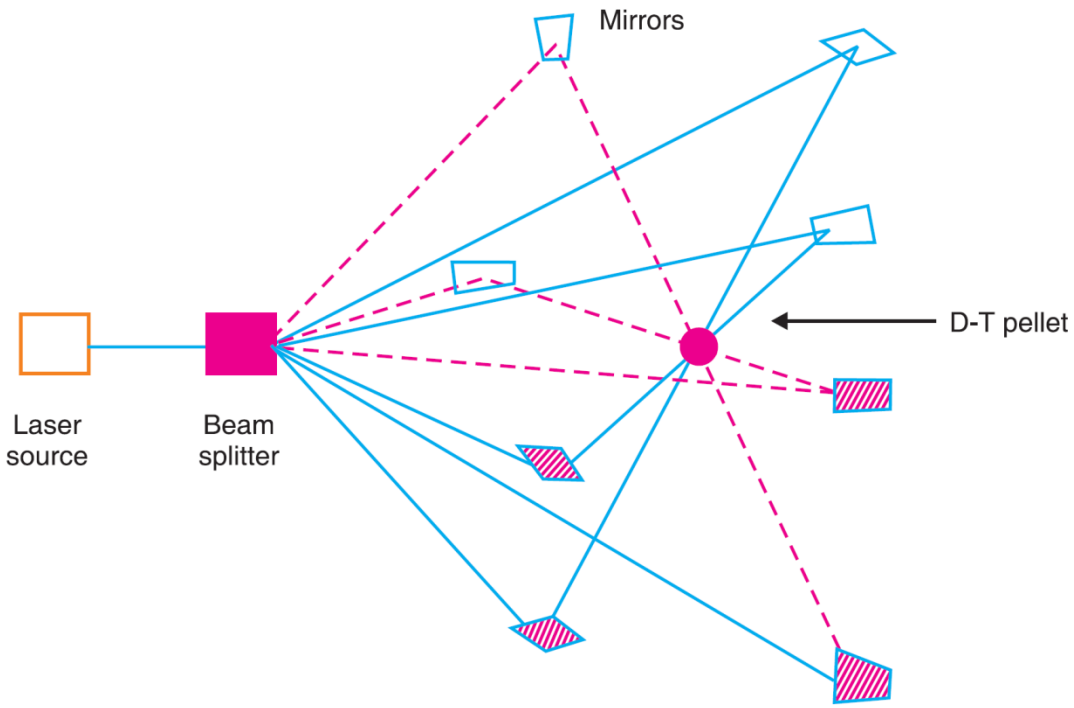


Fig 4.12

Atom Bomb

The principle of fission is made use of in the construction of the atom bomb. An atom bomb consists essentially of two pieces of ${}_{92}U^{235}$ (or ${}_{94}Pu^{239}$) each smaller than the critical size and a source of neutrons. The two subcritical masses of U^{235} in the form of hemispheres are kept apart by using a separator aperture (Fig. 4.13). When the bomb has to be exploded, a third well fitting cylinder of U^{235} (whose mass is also less than the critical mass) is propelled so that it will fit in or fuse together with the other two pieces. Now the total quantity of U^{235} is greater than the critical mass. Hence an uncontrolled chain reaction takes place resulting in a terrific explosion. The explosion of an atom bomb releases tremendously large quantity of energy in the form of heat, light and radiation. A temperature of millions of degrees and a pressure of millions of atmospheres are produced. Such explosions produce shock waves. They are very dangerous because the waves spread radioactivity in air and cause loss of life. The release of dangerously radioactive γ -rays, neutrons and radioactive materials presents a health hazard over the surroundings for a long time. The radioactive fragments and isotopes formed out of explosion adhere to dust particles thrown into space and fall back to earth causing a radiation "*fall-out*", even at very distant places.

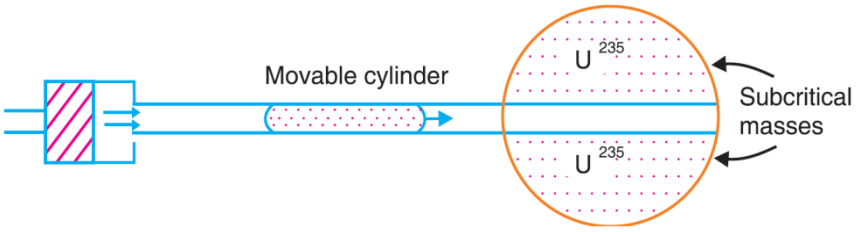


Fig 4.13

Nuclear Fusion

Nuclear fusion. In this process, two or more light nuclei combine together to form a single heavy nucleus. For example, when four hydrogen nuclei are fused together, a helium nucleus is formed. The mass of the single nucleus formed is always less than the sum of the masses of the individual light nuclei. The difference in mass is converted into energy according to Einstein's equation $E = mc^2$

Example. Consider a single helium nucleus formed by the fusion of two deuterium nuclei. Mass of ${}_1H^2$ = 2.014102*u*; mass of ${}_2He^4$ = 4.002604*u*

$$ {}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 $$

The initial mass of 2 deuterium atoms = 2 × 2.014102 = 4.028204 *u*.

Mass of helium atom = 4.002604 *u*.

Decrease in mass = 4.028204 – 4.002604 = 0.025600 *u*.

∴ Energy released = 0.025600 × 931.3 MeV = 23.84 MeV.

Thus the energy released in fusion is 23.84 MeV.

Thermonuclear Reactions

The source of stellar energy is fusion. This suggests that a large amount of energy can be obtained by nuclear fusion. But it is not easy to fuse the light nuclei into a single nucleus. The main difficulty in the fusion of nuclei is the electric force of repulsion between the positively charged nuclei. Fusion is possible when the K.E. of each of the nuclei is large enough to overcome the repulsion. Fusion reactions can take place only at very high temperatures (of the order of 10^7 to 10^9 K). Only at these very high temperatures, the nuclei are able to overcome their mutual Coulomb repulsion and enter the zone of nuclear attractive forces. Hence these reactions are called *thermonuclear reactions.*

A star is able to control thermonuclear fusion in its core because of its strong self-gravity. The thermonuclear reactions in the core of the sun cause high temperatures which generate strong outward pressures; these act against the sun's own gravity, preventing it from contracting, and holding it in equilibrium. The equation of stellar structure, set up by A.S. Eddington, relates the gravitational force in the star to the progressive changes of pressure from its centre outwards, the magnitude of pressure to density and temperature, and the fall of temperature outwards to the flow of energy from the interior to the surface. From these equations, stable models of stars emerge, with central temperatures high enough to start and sustain thermonuclear fusion. The key role, of course, is played by the controlling force of gravity. The large mass of an astronomical system makes gravity the most important factor in determining its behaviour.

Hydrogen bomb

Hydrogen bomb. Hydrogen bomb is a device which makes use of the principle of nuclear fusion. The very high temperature required for an uncontrolled thermonuclear reaction is obtained by the detonation of an atom bomb. In this weapon, hydrogen is the core. The fission bomb produces a very high temperature, at which thermonuclear reactions start resulting in the fusion of hydrogen nuclei to form helium. Greater energy per unit mass is obtained from a hydrogen bomb than from a nuclear fission bomb.

Controlled thermonuclear reactions. A large amount of energy is released in a fraction of a second in a hydrogen bomb. If the thermonuclear reaction could be controlled to take place more slowly, the

energy released can be used for constructive purposes. We know that very high temperatures are needed to bring about a nuclear fusion process. The main problem is to produce such a high temperature and to find a container for the gas which can stand this temperature.

At this temperature the gas is highly ionised and is called *plasma*. One of the severe engineering problems is the design of a "*container*" in which a very hot plasma can be contained under high pressure to initiate a fusion reaction. Since almost any container would melt in the presence of a plasma, attempts are being made to contain and control plasmas trapped in a specially shaped magnetic field (Fig. 4.14). By increasing the field and changing the shape of the field, it is hoped that the plasma in this "*magnetic bottle*" can be raised to the required temperature and pressure for fusion reactions.

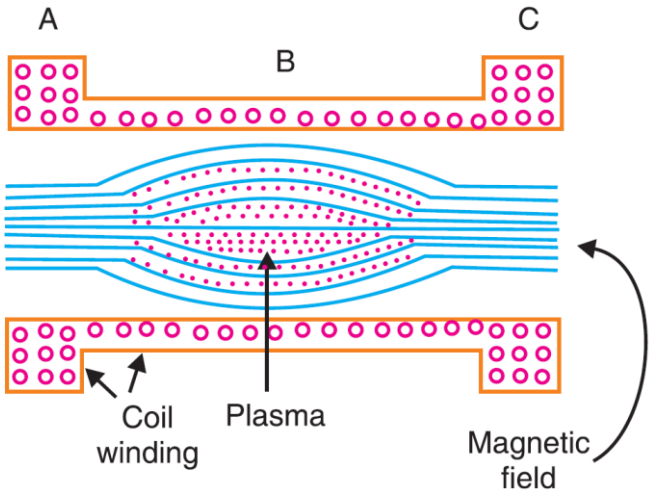


Fig 4.14

Nuclear fusion as an energy source will be a boon to humanity because of the following reasons:

(1) Hydrogen is available everywhere on this planet in various forms.

(2) The lightness of the reactant nuclei makes the energy yield per unit mass of the reacting material much greater than that in nuclear fission process.

(3) A fusion reactor does not leave behind as in fission reactor radioactive waste, the disposal of which poses a tremendous problem.

Example 1. A reactor is developing energy at the rate of 3000 kW. How many atoms of U235 undergo fission per second ? How many kilograms of U235 would by used in 1000 hours of operation assuming that on an average energy of 200 MeV is released per fission ?

SOL. Rate of development of energy by the reactor = 3000 kW = 3×10^6 J s^{-1}

Energy released per fission = 200 MeV = $200 \times 1.6 \times 10^{-13}$ J

$= 32 \times 10^{-12}$ J.

$$\therefore \text{ No. of atoms undergoing fission per second} = \frac{3 \times 10^6}{32 \times 10^{-12}} = 9.4 \times 10^{16}$$

No. of atoms undergoing fission in 1000 hours = $(9.4 \times 10^{16}) \times (1000 \times 60 \times 60)$

$= 3.384 \times 10^{23}$

According to Avogadro's hypothesis, 6.025×10^{26} atoms of U^{235} weight 235 kg.

$$\therefore \text{ Weight of } 3.384 \times 10^{23} \text{ atoms of } U^{235} = \frac{235 \times (3.384 \times 10^{23})}{6.025 \times 10^{26}}$$

$= 0.1321$ kg.

Example 2. A deuterium reaction that occurs in experimental fusion reactor is H^2 (d, p) H^3 followed by H^3 (d, n) He^4. (a) Compute the energy release in each of these. (b) Compute the total energy release per gram of the deuteron used in the fusion. (c) Compute the percentage of the rest mass of deuteron released as energy. (d) Compare U^{235} fission with deuteron fusion as a source of energy release. Given H^2 = 2.014102 u, H^3 = 3.016049 u, H^1 = 1.007825 u, $_0n^1$ = 1.008665 u, He^4 = 4.002604 u and U^{235} = 235.0439 u.

SOL. (*a*) (*i*) The fusion reaction H^2 (*d*, *p*) H^3 is

$$_1H^2 + _1H^2 \rightarrow _1H^3 + _1H^1 + Q$$

Mass decrease in the reaction

$\Delta m = (2.014102 + 2.014102 - 3.016049 - 1.007825)\ u = 0.00433u.$

$\therefore$ Energy released = 0.00433 × 931.3 MeV = 4.032 MeV.

(*a*) (*ii*) The reaction H^3 (*d*, *n*) He^4 is

$$_1H^3 + _1H^2 \rightarrow _2He^4 + _0n^1 + Q$$

$\Delta m = (3.016049 + 2.014102 - 4.002604 - 1.008665)\ u = 0.018842$ u.

Energy released = 0.018842 × 931.3 = 17.58 MeV.

$\therefore$ Total energy release = 4.032 + 17.58 = 21.61 MeV.

(*b*) This total energy release is from the fusion of 3 $_1H^2$ nuclei.

$\therefore$ Energy release per H^2 = 21.61/3 = 7.203 MeV.

$$\text{No. of nuclei in 1 gram of } H^2 = \frac{6.02 \times 10^{23}}{2.014102}$$

$$\therefore \text{Total release of energy from 1 gram of } H^2 = \frac{6.02 \times 10^{23}}{2.014102} \times 7.023 = 2.153 \times 10^{24} \text{ MeV}.$$

(*c*) Energy equivalent of one H^2 nucleus = 2.014102 × 931.3 MeV

Average release of energy per H^2 nucleus = 7.023 MeV.

$$\therefore \left.\begin{array}{l}\text{The percentage of the rest of mass of}\\ \text{deuteron released as energy}\end{array}\right\} = \frac{7.203}{2.014102 \times 931.3} \times 100 = 0.3840\%.$$

(*d*) In U^{235} fission, 200 MeV is released per uranium nucleus.

$$\therefore \left.\begin{array}{l}\text{The percentage of mass energy release in}\\ U^{235} \text{ fission}\end{array}\right\} = \frac{200}{235.0439 \times 931.3} \times 100 = 0.09137\%$$

$$\therefore \frac{\text{Energy release from } H^2 \text{ fusion}}{\text{Energy release from } U^{235} \text{ fission}} = \frac{0.3840}{0.09137} = 4.202.$$

UNIT V – ELEMENTARY PARTICLES

Introduction

After studying the structure of atoms, one gets the impression that perhaps electron, proton and neutron are the only building blocks of matter. Recently the extensive studies made partly on high energy cosmic ray particles and even more, with the help of high energy accelerators, have revealed the existence of numerous new nuclear particles. The subatomic or elementary particles discovered so far, form a long list (around 200). These particles are elementary in the sense that they are structureless, *i.e.*, they cannot be explained as a system of other elementary particles. Table 5.1 shows the elementary particles having relatively long life. Their classification is as follows:

(A) Baryons (or heavy particles). Proton and particles heavier than proton form this group. Proton and neutron are called *nucleons* and the rest are called *hyperons*. Every baryon has an *antiparticle*. If a number, called the baryon number + 1 is assigned to baryons and a number – 1 is assigned to antibaryons, then in any closed system interaction or decay, the baryon number does not change. This is the law of conservation of baryons.

Hyperons is a special class of baryons characterised by a time decay of the order of 10^{-10} seconds and mass value intermediate between those of the neutron and deuteron. Their decay time is very much greater than the time of their formation (10^{-3} sec). It is because of this unsolved problem, that these particles, along with the *K*-mesons, are called *strange particles.* There are four types of hyperons; Lambda, Sigma, Xi and Omega.

(B) Leptons. This group contains electron, photon, neutrino and muon.

(C) Mesons. The rest mass of these particles varies between about 250*me* and 1000*me*. The mesons are the agents of interaction between particles inside the nucleus. Baryons and mesons, are jointly called *hadrons*, and are the particles of strong interaction

Table 5.1 Elementary particles

	Name	*Symbol*	*Rest Energy* M_0 / *MeV*	*Mean lifetime* τ/*s*	*Common decay modes*
Leptons	Photon	γ	0	stable	
	Neutrino	ν_e	0	stable	
		ν_μ	0	stable	
	Electron	$e^{\pm}$	0.511004(2)	stable	
	Muon	$\mu^{\pm}$	105.659 (2)	$2.1994\ (6) \times 10^{-6}$	$e\ \nu\ \bar{\nu}$
Mesons	Pion	$\pi^{\pm}$	139.576 (11)	$2.602\ (2) \times 10^{-8}$	$\mu\nu$
		π^0	134.972 (12)	$0.84\ (10) \times 10^{-16}$	$\gamma\gamma$ (99%) $\gamma e^+ e^-$ (1%)
	Kaon	$K^{\pm}$	493.82 (11)	$1.235\ (4) \times 10^{-8}$	$\mu\ \nu$ (64%) $\pi^{\pm}\ \pi^0$ (21%) 3π (5%)
		K^0	497.76 (16)	50% K_1, 50% K_2	
		K_1		$8.62\ (6) \times 10^{-11}$	$\pi^+\ \pi^-$ (69%) $2\pi^0$ (31%)
		K_2		$5.38\ (19) \times 10^{-8}$	$\pi e\ \gamma$ (39%) $\pi\mu\nu$ (27%) $3\pi^0$ (21%) $\pi^+\ \pi^-\ \pi^0$ (13%)
	Eta	η^0	548.8 (6)		$\gamma\ \gamma$ (38%) $\pi\ \gamma\ \gamma$ (2%) $3\pi^0$ (31%) $\pi^+\ \pi^-\ \pi^0$ (23%) $\pi^+\ \pi^-\ \gamma$ (5%)
Baryons	Proton	$P^{\pm}$	938.256 (5)	Stable	
	Neutron	n	939.550 (5)	$9.32\ (14) \times 10^2$	pev
	Lambda	Λ^0	1115.60 (8)	$2.51\ (3) \times 10^{-10}$	$p\pi^2$ (65%) $n\pi^0$ (35%)
	Sigma	Σ^+	1189.4 (2)	$8.02\ (7) \times 10^{-11}$	$p\pi^0$ (52%) $n\pi^+$ (48%)
		Σ^0	1192.46 (12)	$< 10^{-14}$	$\Lambda\gamma$
		Σ^-	1197.32 (11)	1.49×10^{-10}	$n\pi^-$
	Xi	Ξ^0	1314.7 (7)	$3.03\ (18) \times 10^{-10}$	$\Lambda\pi^0$
		Ξ^-	1321.25 (18)	$1.66\ (4) \times 10^{-10}$	$\Lambda\pi^-$
	Omega	Ω^-	1672.5 (5)	$1.3\ (4) \times 10^{-10}$	$\Xi^0\ \pi^-$, $\Xi^-\ \pi^0$, ΛK^- (?)

(*i*) π-mesons or Pions: π-mesons were discovered in 1947 in the cosmic rays. They can exist in three states: π^+, π^- and π^0. The π^+ and π^- are antiparticles of each other. π^+ and π^- mesons have a rest mass of 273 m_e. The rest mass of π^0 meson is 264 m_e.

The pion is unstable with a mean life of $2.6 \times 10^{-8}\,s$. The charged pions decay into corresponding muon and a μ-neutrino :

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu}$$

The neutral pion has a mean life of 8.7×10^{-17} s and decays into two gamma rays :

$$\pi^\circ \rightarrow \gamma + \gamma$$

μ^+ and μ^- (positive and negative muons) have the same rest mass of 106 MeV $/c^2$ (207 m_e) and the same spin of 1/2. Both decay with a relatively long mean life of 2.2×10^{-6} s into electrons and neutrino-antineutrino pairs.

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu_\mu}$$
$$\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu_e}$$

(*ii*) *K*-Mesons (or Kaons): *K-meson is a heavier class of mesons.* $\boldsymbol{K}^+$ has a rest mass of 966 m_e and mean life of 1.2×10^{-8} s. There are two varieties of neutral *K*-mesons → K_1^0 and K_2^0. Both of them have rest masses of 974 m_e but their mean lives are 9×10^{-11} s and 5×10^{-8} s respectively.

Conservation Laws and Symmetry

There are two types of conservation laws – a) Exact conservation laws b) Approximate conservation laws.

a) Exact Conservation Laws: If these laws violets then the reaction is not allowed in nature. The laws of exact conservation are energy conservation, momentum conservation, angular conservation, charge conservation, charge parity time-reversal conservation.

b) Approximate Conservation laws: If these laws violets then the reaction may or may not be allowed. The laws of approximate conservation laws are isospin, the third component of isospin, strangeness, Charge parity conservation, Charge conjugation and time reversal conservation.

A very important set of conservation laws is related to symmetries involving parity (*P*), charge conjugation (*C*), and time reversal (*T*).

Conservation of parity. Parity relates to the symmetry of the wave function that represents the system. If the wave function is unchanged, when the coordinates (*x*, *y*, *z*) are replaced by (– *x*, – *y*, – *z*), then the system has a parity of + 1. If the wave function has its sign changed, when the coordinates are reversed, then the system has a parity of – 1. If we write ϕ (*x*, *y*, *z*) = P ϕ (– *x*, – *y*, – *z*), we can regard *P* as a quantum number characterizing ϕ whose possible values are + 1 and – 1. During a reaction in which parity is conserved, the total parity number does not change. Changing the coordinates (*x*, *y*, *z*) into (– *x*, – *y*, – *z*) converts a right-handed coordinate system into a left-handed coordinate system. In terms of symmetry, the meaning of conservation of parity is that in any situation where parity is conserved, the description of the reaction will not be changed if the word "left" is changed to the word "right" and vice versa. This means that such reactions can provide no clue that will distinguish between the directions right and left. Prior to 1956 it was believed that all reactions in nature obeyed the law of conservation of parity. However, Yang and Lee pointed out that in reactions involving the weak

interaction, parity was not conserved, and that experiments could be devised that would absolutely distinguish between right and left. Indeed parity conservation is found to hold true only in the strong and electromagnetic interactions.

Charge conjugation symmetry. Charge conjugation is the act of symmetry operation in which every particle in a system is replaced by its antiparticle. If the anti-system, or antimatter counterpart exhibits the same physical phenomena, then charge parity (*C*) is conserved. For example, if in a hydrogen atom, the proton is replaced by an antiproton and the electron is replaced by a positron, then this antimatter atom will behave exactly like an ordinary atom, if observed by people also made of antimatter. In fact *C* is not conserved in the weak interaction.

Time reversal symmetry. Time parity *T* describes the behaviour of a wave function when *t* is replaced by – *t*. The symmetry operation that corresponds to the conservation of time parity is time reversal. Time reversal symmetry implies that the direction of time is not significant, so that the reverse of any process that can occur is also a process that can occur. In other words, if symmetry under time reversal holds, it is impossible to establish by viewing it whether a motion picture of an event is being run forward or backward. Prior to 1964, time parity *T* was considered to be conserved in every interaction. It was discovered in 1964 that one form of the K^0 kaon can decay into $\pi^+ + \pi^-$,which violates the conservation of *T*. The symmetry of phenomena under time reversal thus does not seem to be universal.

Combined inversion of CPT. The combined symmetry operation in which the antimatter mirror image of a system is run in reverse allows a test of *CPT* invariance. All the evidence supports the conservation of *CPT*. The conservation of *CPT* means that for every process there is an antimatter mirror-image counterpart that takes place in reverse. This particular symmetry seems to hold for all interactions, even though its component symmetries sometimes fail individually.

The Fundamental Interactions

Four kinds of interaction between elementary particles can account for all known processes in the physical universe on all scales of size. In Table 5.2 the four basic interactions are summarised.

Table 5.2 The four fundamental interactions

Interaction	*Particles Affected*	*Range*	*Relative Strength*	*Particles Exchanged*
Strong	Hadrons	$\sim 10^{-15}$ m	1	Mesons
Electro-magnetic	Charged Particles	∞	$\sim 10^{-2}$	Photons
Weak	Hadrons and leptons	$\sim 10^{-17}$ m	$\sim 10^{-13}$	Intermediate bosons
Gravitational	All	∞	$\sim 10^{-40}$	Gravitons

(1) **Strong interaction.** A familiar example of strong interaction is the forces which hold nucleons together (nuclear forces) in the atomic nucleus. The strong nuclear interaction is independent of the electric charge. The range of these interactions is about 10^{-15} m. Time interval of such an interaction is roughly 10^{-23} s.

(2) **Electromagnetic interaction.** It operate on all charged particles. Thus electromagnetic interactions are charge dependent (attractive as well as repulsive). The range is infinite and the interaction works through the *photon*. The formation of electron-positron pair from γ-ray is an example of electromagnetic interaction.

(3) **Weak interaction.** All strong interactions take place in times of about 10^{-23} s (characteristic time). Yet is has been observed that some of the resulting particles, although energetically unstable, suffer no decay until a time 1013 times greater than 10^{-23} is reached. That is, their decay takes place in time of about 10^{-10} s. For example, β-decay of radioactive nuclei does not take place until a time 10^{13} times greater than that involved in strong interactions has approached. Had there been strong nuclear or electromagnetic interactions, there would have been no such delay in the decay process. Therefore, this delay in the decay process suggests that either these particles are not subjected to strong interacting forces or there is some new conservation law or prohibition which forbids the decay. But since most of the particles involved are subjected to either nuclear force or have electric charge or both, there must be some rule which stops the process. But eventually the decays do happen, there must be some other type of interaction as predicted by Fermi to explain β-decay. Since particles take long time to respond to such an interaction, force involved must be very weak proton electron positron compared with strong nuclear forces. The range of such an interaction is less than 10^{-17} m. The characteristic time of this interaction is $\approx 10^{-10}$ s. Yukawa, in 1938, suggested that there should be a field quantum for the weak interaction, corresponding to the photon and pion. This so called *intermediate vector boson*. It has not been experimentally detected as a free particle. The weak interaction is responsible for the decay of strange and non-strange particles and for non-leptonic decays of strange particles.

(4) **Gravitational interaction.** It is the weakest of the four types of interactions. It has infinite range. Although gravity has a measurable influence on macroscopic bodies, its interaction with subnuclear particles is very small. Gravitation can be explained in terms of the interactions of '*gravitons*'. Their mass must be zero, and therefore, their velocity must be that of light. As the gravitational field is extremely weak, the gravitons can not be detected in the laboratory. Of the four basic forces, only gravitational force is universal. Weak forces affect every particle other than photon. Electromagnetic force is confined to charged particles. Strong forces are the most selective and they serve as the criteria for classifying all known particles other than photons into two broad categories, the leptons and the hadrons. Leptons and photons are light particles and do not feel the strong forces. But hadrons feel the strong forces and participate in strong interactions besides taking part in other types of interactions as well. Thus the proton is a strongly interacting nuclear constituent and is therefore, a hadron. At the same time, because of its charge and mass, it must also experience electromagnetic and gravitational forces. The fact that it can be created by β-decay of neutron shows that the proton is involved in weak interactions as well.

Particles and Anti-particles

Electron and Positron. The positron and the electron are said to be antiparticles. They have the same mass and the same spin but opposite charge. They annihilate each other with the emission of photons, when they come in contact with each other. The existence of an antiparticle for the electron was actually predicted by Dirac, because of a symmetry of the equations of the relativistic quantum theory of the electron. Positron was discovered by Anderson in 1932.

Proton and antiproton. Dirac's theory, anticipating the positron, could be interpreted as implying a particle identical to the proton, except for a negative charge. The existence of this particle, the

antiproton, was established in 1955 by Segre, Chamberlain, and their collaborators. Antiprotons were produced by bombarding protons in a target with 6-GeV protons, thereby inducing the reaction

$$p + p\,(+\text{energy}) \rightarrow p + p + p + \overline{p}$$

The K.E. of the bombarding proton is converted to a proton-antiproton pair plus the K.E. of the four residual particles. Antiprotons interact strongly with matter and annihilate with protons. In a typical annihilation reaction, the rest mass of the annihilating pair appears as five pions and their K.E.

$$p + \overline{p} \rightarrow \pi^+ + \pi^- + \pi^+ + \pi^- + \pi^0 .$$

Neutron and antineutron. The antiparticle of neutron, *antineutron*, was discovered in 1956 by Cork, Lamberton and Wenzel. The nature of the antineutron is not very well known. Both neutron and antineutron have zero charge and the same mass. However, since neutron is supposed to have a certain internal charge distribution, it is expected that the antineutron has an internal charge distribution opposite to that of the neutron. Antineutron is quickly annihilated, either by a proton or a neutron, usually with the production of several pions. If an antineutron is not annihilated by a nucleon, it decays by the reaction

$$\overline{n} \rightarrow \overline{p} + \beta^+ + \nu .$$

Neutrino and antineutrino. It is possible to avoid violations of the law of conservation of energy, linear and angular momenta and spin by assuming the existence of a particle, called a *neutrino*. It has *charge zero, spin* 1/2, *zero rest mass* (*rest mass very small in comparison with that of the electron*) *and magnetic moment smaller than* 10^{-8} *Bohr magneton or nearly zero.* The neutrino has a finite energy and momentum in flight. It travels with the speed of light *c*. It does not cause ionization on passing through matter. The antiparticle of neutrino is *antineutrino*. The distinction between the neutrino ν and antineutrino n– is a particularly interesting one. The spin of the neutrino is opposite in direction to the direction of its motion; viewed from behind, the neutrino spins-counterclockwise. But the spin of the antineutrino is in the same direction as its direction of motion; viewed from behind, it spins clockwise. Thus the neutrino moves through space in the manner of a left-handed screw, while the antineutrino does so in the manner of a right-handed screw. Thus a neutrino possesses a "left-handed" helicity; the antineutrino possesses a "right-handed" helicity, *i.e., a neutrino and antineutrino differ only in the sense of their helicity*. It is customary to call the particle accompanying a positron a neutrino, ν, while that accompanying an electron is called an antineutrino, n–. It is important to remember that positrons and electrons are never ejected alone. It is clear from experiments that neutrinos emitted in β decay (positron decay and *K*-capture) have a negative helicity, *i.e.*, they are longitudinally polarized with their spin axes antiparallel to their direction of travel.

$$\mathbf{H} = +1 \text{ for } \overline{\nu} \; ; \; \mathbf{H} = - \text{ for } \nu .$$

Because of its lack of charge and magnetic moment, a neutrino has essentially no interaction with matter, except that which leads to inverse β decay. This interaction is extremely weak. The cross

section for this process is only $\sigma \approx 10^{-48}$ m^2 $\approx 10^{-20}$ *barn*. Matter is almost totally transparent to neutrinos.

Resonance Particles

Almost all of the known particles are unstable. They decay, spontaneously falling apart into several other particles. The only absolutely stable particles are the electron, the proton, the photon, and the neutrinos. The decay of a kaon and that of a lambda particle are some examples of decays. Each of these particles lives for about 10^{-10} s from the instant of production to the instant of decay. By the standards of high-energy physics, a time interval of 10^{-10} s is a rather long time interval. If a particle lives that long, physicists can do experiments on it and with it. In the terminology of high-energy physics, any particle with a lifetime of 10^{-10} or even 10^{-14} s is regarded as stable. However, during the past decade or so there has been a profusion of new particles. These new particles are characterised by a life time of about 10^{-23} s showing that they are extremely unstable. In order to avoid answering the question whether they are really fundamental particles or not, physicists have started calling them '*resonances*' or resonance particles. These are all strongly interacting. The word resonance has been chosen to indicate them because they are recognised by resonance peaks in the normal energy spectrum of an event.

Production. The short-lived particles cannot be detected directly, but their existence can be deduced from circumstantial evidence. Consider the case of the short-lived lambda particle $\Lambda(1520)$, found in collisions between kaons and protons. If a beam of negative kaons impacts on the protons in a bubble chamber, a variety of reactions will occur. Two of these are :

$$K^- + p \rightarrow \Sigma^0 + \pi^0 \qquad ...(1)$$

$$K^- + p \rightarrow \Lambda + \pi^+ + \pi^- \qquad ...(2)$$

The reaction cross-section for each of the reactions is some function of energy. Fig. 40.4 shows how the cross-sections vary as function of the centre-of-mass energy. There is a local maximum in the cross-sections at a centre-of-mass energy of 1520 MeV. At this energy *all* the cross-sections have a peak. So, all the maxima are due to the same cause. In each case, the colliding particles merge to create a new particle :

$$K^- + p \rightarrow \Lambda\,(1520) \qquad ...(3)$$

This new particle subsequently decays into one or another of the pairs of particles that emerge from the collision :

$$K^- + p \rightarrow \Lambda\,(1520) \rightarrow \Sigma^0 + \pi^0 \qquad ...(4)$$

$$\rightarrow \Lambda + \pi^+ + \pi^- \qquad ...(5)$$

According to Eq. (3), the production of a lambda will occur only if the energy of the kaon plus the mass-energy of the proton add up to the mass-energy of the lambda. The lambda particle in the reactions (4) and (5) plays a role analogous to that of the compound nucleus in nuclear reactions. Short-lived particles such as the lambda are often called *resonances*. Thus one speaks of the Λ (1520) as a K^- *p* resonance.

Decay Time Calculation. The plots of Fig. 5.1 show that the cross-section is not only large when the centre-of mass energy is exactly 1520 MeV, but is also quite large at energies somewhat below and somewhat above 1520 MeV. This indicates that there is an uncertainty in the energy for the production of a lambda. The magnitude of this uncertainty is about ± 16 MeV. The uncertainty in the production energy implies an uncertainty in the mass-energy of the lambda. The mass is uncertain by ± 16 MeV/*c*2.

$$M_{\Lambda\,(1520)} = 1520\ \text{MeV}/c^2 \pm 16\ \text{MeV}/c^2.$$

We can estimate the lifetime from the uncertainty in energy :

$$\Delta t \simeq \frac{\hbar}{\Delta E} \simeq \frac{1{\cdot}05 \times 10^{-34}\ \text{Js}}{16\,\text{MeV}} \simeq 4 \times 10^{-23}\ \text{s}$$

Such extremely short life-times can be estimated only from the uncertainty principle, since an unstable particle does not travel far enough to leave a measurable track in a detector, say a bubble chamber, before decaying.

Detection of Neutrino.

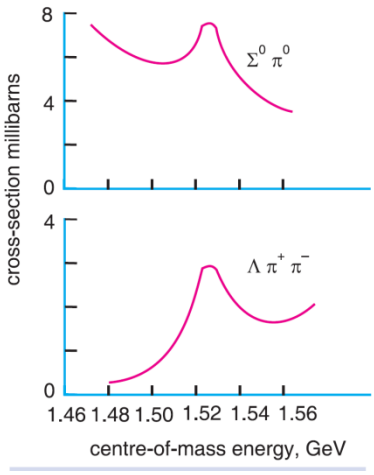


Fig 5.1

Hypernuclei

Hypernuclei are nuclei in which at least one nucleon is replaced by a hyperon (a baryon that contains one or more strange quarks.)

The Quark Model

Murray Gell-Mann and G. Zweig proposed the quark model in 1964. This theory is based on the idea that the hadrons are built up from a limited number of "fundamental" units, which have acquired the name quarks. The original three quarks were labeled *u* (for "*up*"), *d* (for "*down*") and *s* (for "*strange*").

u quark has electric charge $+\frac{2}{3}e$ and strangeness 0.

d quark has electric charge $-\frac{1}{3}e$ and strangeness 0.

s quark has electric charge $-\frac{1}{3}e$ and strangeness – 1.

Each quark has a baryon number of $B = 1/3$.

Each quark has an antiquark associated with it ($\bar{u}$,$\bar{d}$,and $\bar{s}$). The magnitude of each of the quantum numbers for the antiquarks has the same magnitude as those for the quarks, but the sign is changed.

Compositions of hadrons according to the quark model

Hadrons may be *baryons* or *mesons*. A baryon is made up of three quarks. For example, the proton is made up of two *u* quarks and a *d* quark (*uud*). For these quarks, the electric charges are +2/3, + 2/3, and – 1/3, for a total value of + 1. The baryon numbers are + 1/3, + 1/3 and + 1/3, for a total of + 1. The strangeness numbers are 0, 0 and 0 for a total strangeness of 0. All are in agreement with the quantum numbers for the proton. Fig. 5.2 shows quark models of the proton, antiproton, neutron and antineutron. Electric charges are given in units of *e***.**

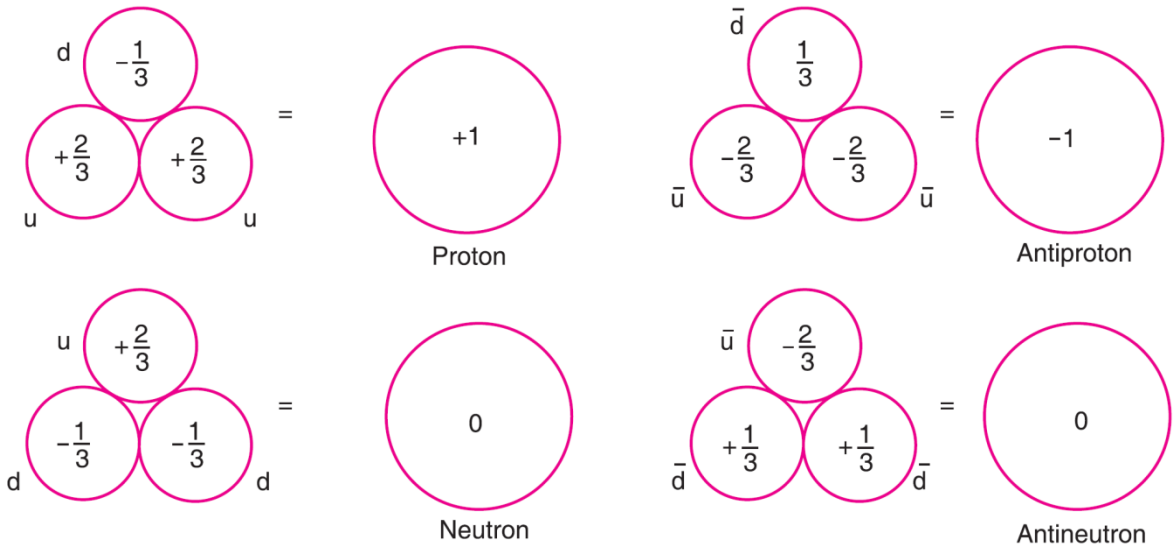


Fig 5.2

A meson is made up of one quark and one antiquark. For example, the π **+ meson** is the combination of a *u* quark and a *d* antiquark (*ud*). Electric charges of these quarks are + 2/3 and + 1/3 for a total of + 1. The baryon numbers are + 1/3 and – 1/3 for a total baryon number of 0. The strangeness numbers are 0 and 0 for a total of 0. All of these are in agreement with the quantum numbers for the pi-meson. Quarks all have spins of 1/2, which accounts for the observed half-integral spins of baryons and the 0 or 1 spins of mesons.

All known hadrons can be explained in terms of the various quarks and their antiquarks. Table 5.3 shows the quark contents of five hadrons and how they account for the observed charges, spins, and strangeness numbers of these particles.

Table 5.3 Compositions of some hadrons according to the Quark Model

Hadron	*Quark content*	*Baryon number*	*Charge, e*	*Spin*	*Strangeness*
π^+	$u\bar{d}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	$\uparrow\downarrow=0$	$0+0=0$
K^+	$u\bar{s}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	$\uparrow\downarrow=0$	$0+1=+1$
P^+	uud	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$+\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$	$\uparrow\uparrow\downarrow=\frac{1}{2}$	$0+0+0=0$
n^0	ddu	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}=0$	$\downarrow\downarrow\uparrow=\frac{1}{2}$	$0+0+0=0$
Ω^-	sss	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=-1$	$\uparrow\uparrow\uparrow=\frac{3}{2}$	$-1-1-1=-3$

Coloured quarks and gluons : There were problems with the quark model, one of them being $\Omega-$ hyperon. It was believed to contain three identical *s* quarks (*sss*). This violates the Pauli exclusion principle, that prohibits two or more fermions from occupying identical quantum states. The proton, neutron, and others with two identical quarks would violate this principle also. We can resolve this difficulty by assigning a new property to the quarks. We can regard this new property as an additional quantum number that can be used to label the three otherwise identical quarks in the Ω^-. If this additional quantum number can take any one of three possible values, we can restore the Pauli principle by giving each quark a different value of this new quantum number, which is known as *colour*. The three colours are labeled red (*R*), blue (*B*), and green (*G*). The $\Omega-$ for example, would then *s*R *s*B *s*G. The antiquark colours are antired (*R*) antiblue (*B*) and antigreen (*G*).

An essential component of the quark model with colour is that *all observed meson and baryon states are* "*colourless*", *i.e.*, either colour anticolour combinations in the case of mesons, or equal mixtures of *R*, *B* and *G* in the case of baryons. Since hadrons seem to be composed of quarks, the strong interaction between hadrons should ultimately be traceable to an interaction between quarks. The force between quarks can be modeled as an exchange force, mediated by the exchange of massless spin – 1 particles called gluons. Eight gluons have been postulated. The field that binds the quarks is a *colour field. Colour is to the strong interaction between quarks as electric charge is to the electromagnetic interaction between electrons.* It is the fundamental strong "charge" and is carried by the gluons. The gluons must therefore be represented as combinations of a colour and a possibly different anticolour

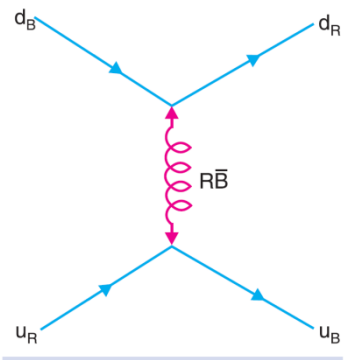
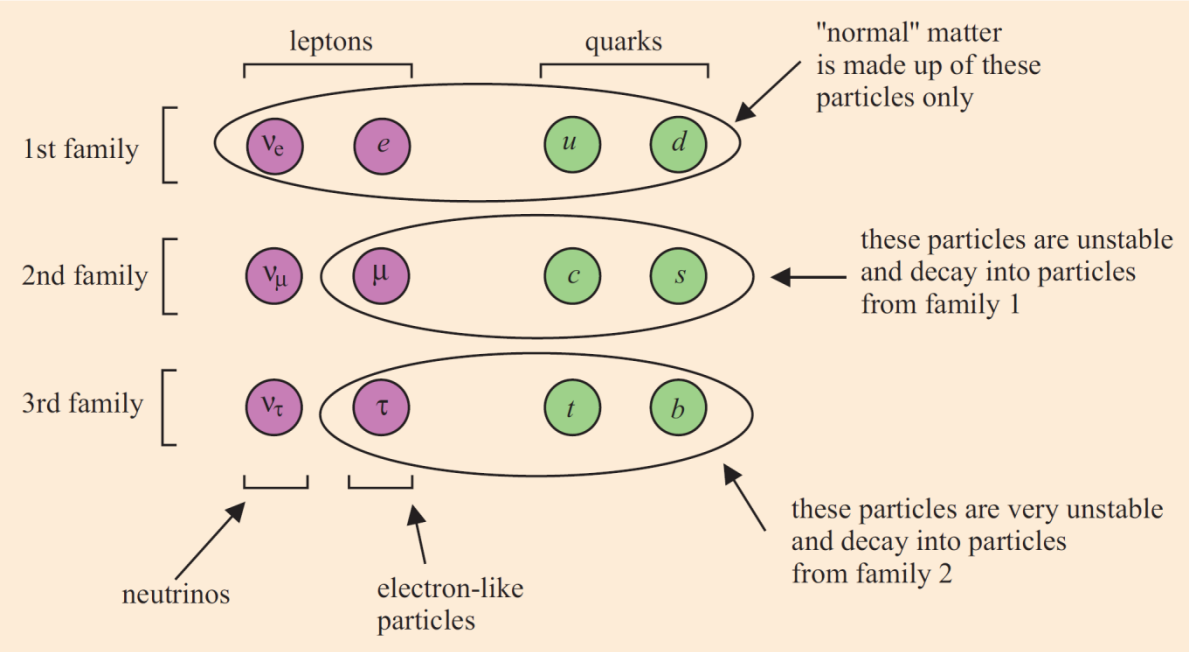


Fig 5.3

The gluons are massless and carry their colour-anticolour properties just as other particles may carry electric charge. For example, Fig. 5.3 shows a gluon *RB–* being exchanged by red and blue quarks. In effect the red quark emits its redness into a gluon and acquires blueness by also emitting antiblueness. The blue quark, on the other hand, absorbs the *RB–* gluon, cancelling its blueness and acquiring a red colour in the process.

Charm, Bottom, and Top. In 1970, Glashow, Iliopoulis, and Maiani proposed the existence of a fourth quark, called *c* or *charmed* quark. The charmed quark was suggested to explain the suppression of

certain decay processes that are not observed. With only three quarks, the processes would proceed at measurable rates and should have been observed. The charm quark has a charge of 2/3 *e* strangeness 0 and a charm quantum number of + 1. Other quarks have 0 charm.



In 1977, a new particle was discovered at Fermi Lab that provided evidence for yet another quark. This particle, called the upsilon-meson, was thought to be made up of the new quark called *b* (for bottom or beauty) along with the associated antiquark $\bar{b}$. *b* quark has electric charge –1/3*e*.

Because quarks seem to come in pairs, it is expected that there is a partner to the *b* quark, called *t* (for top, if *b* = bottom, or truth, if *b* = beauty). It has a charge of + 2/3*e*.

Three generations of quarks and leptons. Both leptons and quarks appear to come in three generations of doublets, with all particles having spin 1/2. Table 5.4 shows the properties of the three generations of quarks and leptons. The first generation contains two leptons, the electron and the electron neutrino, and two quarks, up and down. All the properties of ordinary matter can be sunderstood on the basis of these particles. The second generation includes the muon and muon neutrino and the charm and strange quarks. These particles are responsible for most of the unstableparticles and resonances created in high energy collisions. The third generation includes the tau and the tau-neutrino and the top and bottom quarks.

Table 5.4 **Properties of the three generations of Quarks and Leptons**

Generation	*Quark*	*Symbol*	*Charge, e*	*Strangeness*	*Charm*
1	Up	u	$+\frac{2}{3}$	0	0
	Down	d	$-\frac{1}{3}$	0	0
2	Charm	c	$+\frac{2}{3}$	0	+ 1
	Strange	s	$-\frac{1}{3}$	– 1	0
3	Top	t	$+\frac{2}{3}$	0	0
	Bottom	b	$-\frac{1}{3}$	0	0

Generation	*Lepton*	*Symbol*	*Charge, e*
1	Electron	e^-	– 1
	e-Neutrino	ν_e	0
2	Muon	μ^-	– 1
	μ-Neutrino	ν_μ	0
3	Tau	τ^-	– 1
	τ-Neutrino	ν_τ	0